Hedging with Interest Rate Swap

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***Abstract*—Despite the importance played by Interest Rate Swaps (IRS), it appears that sounding analyzes related to the hedging of portfolios made by swaps is not clear in the financial literature.**

We provide here the analysis corresponding to a parallel shift of the interest rate. The suitable swap sensitivities to make use in hedging and risk management obtained here may be seen as some generalization of the well known bond duration and convexity in the swap framework.

Our present results might provide a support for practitioners, using portfolio of swaps and/or bonds, in their hedge decision-making.

***Index Terms*—Hedging, optimization, zero-coupon, swap.**

I. Introduction

Interest Rate Swaps (IRS) appear to be instruments largely used by market participants (companies, local governments, financial institutions, traders ...) for many purposes including debt structuring, regulatory requirements and risk management. According to the BIS June 2011 statistics, the Interest Rate Swap (IRS) represents 78.25% of OTC derivatives while the corresponding equity part is just about 0.97%.

Despite this market importance played by IRS, it appears that sounding analyzes related to the hedging of portfolios made by swaps is not clear in the financial literature. To partially fill this lack, we provide here the analysis corresponding to a Parallel Shift (referred in the sequel as (PS)) of the interest rate. Though such an underlying assumption is little bit less realistic, both practical and theoretical reasons lead to grant a consideration to this particular situation.

Some of the arguments are presented in our (lengthy) working paper [1], where we have already analyzed the portfolio hedging using swaps and bonds. Parts of our findings are summarized and reported here. In our numerical illustrations we consider the hedge of a swap portfolio by another swap portfolio, a case which has not been considered before. The suitable swap sensitivities to make use in hedging and risk management are obtained here as a byproduct of our analyses. They may be seen as generalizing the well-known bond duration and convexity [2]-[3] in the swap framework. These obtained sensitivities are in line with the bond situation, for which the need to take into account both the passage of time and horizon hedging are analyzed in [4] and [5].

Our aim in writing this paper is to provide a theoretical support which shed light practitioners in their decision-making related to the hedge of a position sensitive to interest rate and by using a portfolio made by swaps (and/or bonds). For the time being, there are various broker advertisements and leaflets about switching to alternative instruments (as VIX futures, inverse ETF, Swap future ...) for the hedging purpose instead of just using a classical bond portfolio. However the arguments used in these leaflets are essentially based on (particular) numerical situations which are certainly attractive but unfortunately do not reflect all other cases which may arise in reality. Systematic analysis of the portfolio hedging mechanism, as performed here, cannot really fully describe over commercial flyers.

Our present project is essentially focused on the hedge of a position sensitive to the interest rate by a portfolio of swaps. The use of a bond portfolio as a hedging instrument has been investigated in [5]. It may be noted that the hedge with a bond future was previously studied in [6] and empirically investigated in [7]. Here we do systematic analyses of the hedging mechanism in the sense that they are essentially based on the portfolio instrument characteristics and, in contrast with various academicals papers and commercial leaflets related to hedging, they do not lean on particular historical data. Our results provide an approach and formulas which may be directly implemented in order to get the suitable hedge ratio and corresponding hedging error estimates for any given portfolio of swaps. Of course the interest rate curve, at the hedge horizon, is assumed here to make a parallel shift belonging to some closed finite interval. Though this last seems to be a restrictive assumption, any realistic interest rate curve movement is always inside some band which may be determined based on the market view. It means that we have derived here some sort of robust hedging approach in the sense that it avoids to use involved dynamical stochastic model for the interest rate.

In Section 2, we make a survey of our results, whose technical details are presented in [1]. Then a numerical illustration is displayed in the next Section 3.

II. Survey of Our Results

After recalling features on Zero-Coupon-Bond and Interest Rate Swap in 2.1, we present in Subsection 2.2 the underlying idea to the hedge of a portfolio of swaps by another portfolio. Our main contribution is on the derivation of the sensitivities and the associated optimization.

A. Zero-Coupon and Interest Rate Swap

The present time is denoted by t. By $P(t,T)$ we mean the time-t price of a Zero-Coupon-Bond which, can be seen as an instrument paying one unit of the currency to its holder at the

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maturity T, where $t < T$ Commonly it is taken that

$$P(t,T) = \exp[-y(t;T-t)(T-t)] \tag{1}$$

where $T-t$ is the time-to-maturity. The nonnegative real number $y(t;T-t)$ represents the interest rate applied at time t for this time-to-maturity and very often known under the name of yield-to-maturity.

A plain vanilla Interest Rate Swap is a contract between two counterparties. The first agrees to pay to the second, during a given period of time, regularly a cash flow equal to the interest corresponding to a predetermined fixed rate on the contractual notional principal. In return, the first counterparty receives an interest at floating rate on the same notional principal and for the same period of time. It may be seen [1] that, the time-t value for such a swap (with respect to the point of view of the counterparty paying the fixed rate) is given by

$$value\_Swap_t = notional \times \begin{pmatrix} P(t,t_1)\{y(t_0;t_1) - rate\_Swap_t\}\tau(t,t_1) \\ +\{P(t,t_1) - P(t,t_M)\} \\ -rate\_Swap_t \times \sum_{i=2}^{M} P(t,t_i)\tau(t_{i-1},t_i) \end{pmatrix} \tag{2}$$

.

where

$$0 \le t_0 \le t \le t_1 < \ldots < t_i < \ldots t_M$$

such that $t_1 \ldots t_i \ldots t_M$ correspond to the cash-flow time-payments. Here $\tau(t_{i-1},t_i)$ denotes the annual measure of the time-elapsed between t_{i-1} and t_i. For a semi-annual frequency one has $\tau(t_{i-1},t_i) \approx 0.5$. By rate_Swap we mean the contractual predetermined rate, such that at the contract time inception the swap has a zero market value.

The swap market value, as in (2) are one things, but for the position management and hedging the change of the market value matters. Therefore for the (future) time-period $(t, t+\delta)$, let us set

$$\begin{aligned} &change\_value\_Swap_{t,t+\delta}(.) = \\ &value\_Swap_{t+\delta}(.) - value\_Swap_t \end{aligned} \tag{3}$$

To simplify we only consider the case $t+\delta < t_1$ such that no payment takes place during $(t,t+\delta)$ When such an assumption is not satisfied then at least an effective cash-flow is paid or received and the analysis becomes little bit complicated. The assumption used here relies on the fact that in practice the horizon under consideration is preferably short enough in order the associated projected view to be more and less credible. However the real hedging horizon may be for a longtime, and consequently it is usual among the practitioners to roll their hedging positions. It means that it is important to have at a disposal an accurate analysis for the single-period hedging and it is exactly our main focus in this paper. The explicit value of change value of the Swap during the period $(t,t+\delta)$ may be written as

$$change\_value\_Swap_{t,t+\delta}(.) = notional \times \begin{pmatrix} -\{y(t_0;t_1) - rate\_Swap_t\}P(t,t_1)\delta \\ +\left(1+\{y(t_0;t_1) - rate\_Swap_t\}\tau(t+\delta,t_1)\right)\begin{pmatrix} P(t+\delta,t_1)(.) \\ -P(t,t_1) \end{pmatrix} \\ -\{P(t+\delta,t_M)(.) - P(t,t_M)\} \\ -rate\_Swap_t \times \sum_{i=2}^{M}\left(P(t+\delta,t_i)(.) - P(t,t_i)\right)\tau(t_{i-1},t_i) \end{pmatrix} \tag{4}$$

With this last expression the swap market value change during the time-period $(t,t+\delta)$ arises as a linear combination of changes of zero-coupon bonds with various maturities t_i 's. It means that we have to make use of features concerning zero-coupon changes

$$P(t+\delta,t_i)(.) - P(t,t_i)$$

For such a purpose, a model for the future evolution of the interest rate is needed. In this paper we will focus on the common hypothesis ″Parallel Shift (PS) of the yield curve″ at the future time $(t+\delta)$ which is described by

$$y(t+\delta;\tau)(.) = y(t;\tau) + \varepsilon(.)$$

where $\varepsilon(.) = \varepsilon(.,t,\delta)$ In this last, we mean that the shift ε depends on the present time t and horizon δ. The strong fact here (and likely less realistic) is that the shift does not depend on the maturity τ .

B. The Hedging Mechanism and Sensitivities $P(t,T)$

Let us denote by V_t the present time t-value of a portfolio assumed to be sensitive to the interest rate which is made by swaps or/and bonds. At the future time horizon $t+\delta$ this portfolio may suffer from a loss, in the sense that $V_{t+\delta} < V$. So to try to maintain the (future) value $V_{t+\delta}(.)$ to be close to V_t , the portfolio manager has to put in place a hedging technique. The idea relies on using another portfolio, referred in the sequel as a hedging portfolio (or instrument), such that this last would lead to a nonnegative profit compensating the loss on the initial portfolio. Therefore instead of the absolute change

$$V_{t+\delta}(.) - V_t = P\&L\_naked\_portfolio_{t,t+\delta}(.) \tag{5}$$

associated with the initial naked portfolio, at the horizon $t+\delta$, the change for the covered portfolio is given by

$$\begin{aligned} &P\&L\_covered\_portfolio_{t,t+\delta}(.) = V_{t+\delta}(.) - V_t \\ &+ P\&L\_hedging\_instrument_{t,t+\delta}(.) \end{aligned} \tag{6}$$

The hedging portfolio H is assumed at time-t to have the value

$$H_t = \sum_{i^{**}=1}^{I^{**}} H^{**}_{t,i^{**}} n^{**}_{i^{**}} - \sum_{i^{*}=1}^{I^{*}} H^{*}_{t,i^{*}} n^{*}_{i^{*}} \tag{7}$$

It means that H is made by I^{**} types of instruments $H^{**}_{t,i^{**}}$ in long positions and I^{*} types of instruments $H^{*}_{t,i^{*}}$ in short positions. For a given type i^{**} (resp. i^{*}), we make use of $n^{**}_{i^{**}}$ (resp. $n^{*}_{i^{*}}$) number of instruments $H^{**}_{t,i^{**}}$ (resp. $H^{*}_{t,i^{*}}$). The Profit&Loss corresponding to the use of the hedging instrument is (roughly) given by

$$\begin{aligned} &P\,\&\,L\_hedging\_instrument_{t,t+\delta}(.) \\ &=\{H_{t+\delta}(.)-H_t\}-\cos t\_H_t \end{aligned} \tag{8}$$

such that

$$\begin{aligned} &P\,\&\,L\_\text{cov}ered\_portfolio_{t,t+\delta}(.)= \\ &V_{t+\delta}(.)-V_t+\sum_{i^{**}=1}^{I^{**}}\{H^{**}_{t+\delta,i^{**}}-H^{**}_{t,i^{**}}\}n^{**}_{i^{**}} \\ &-\sum_{i^{*}=1}^{I^{*}}\{H^{*}_{t+\delta,i^{*}}-H^{*}_{t,i^{*}}\}n^{*}_{i^{*}}-\cos t\_H_t \end{aligned} \tag{9}$$

where

$$\begin{aligned} &\cos t\_H_t=\left\{\frac{1}{P(t,t+\delta)}-1\right\}\times \\ &\left(\sum_{i^{**}=1}^{I^{**}}\{v_0^{**}+v^{**}|h^{**}_{t,i^{**}}|\}N^{**}_{i^{**}}n^{**}_{i^{**}}-\sum_{i^{*}=1}^{I^{*}}\{v_0^{*}+v^{*}|h^{*}_{t,i^{*}}|\}N^{*}_{i^{*}}n^{*}_{i^{*}}\right) \end{aligned} \tag{10}$$

With $v_0^{**}, v^{**}, v_0^{*}, v^{*}$, are fixed constants such that $0 \le v_0^{**}, v_0^{*} < 1$, and $0 < v^{**}, v^{*} < 1$ The numerical values of these constants depend on the market practice under consideration. In (10), we have used the fact that the instrument value, $H^{**}_{t,i^{**}}$ is the product of its notional $N^{**}_{i^{**}}$ with its one unit value $h^{**}_{t,i^{**}}$. For an instrument satisfying $h^{**}_{u,i^{**}} \ne 0$ during its life-time, as in the case of a (risk credit free) bond for example, the corresponding cost at time t is very often defined a $v^{**}H^{**}_{t,i^{**}}$; so that here one can take $v_0^{**}=0$. The introduction of v_0^{**} nd v_0^{*} relies on the fact that for some instruments as a swap, one can have that the corresponding market value satisfies $H^{**}_{t,i^{**}}=0$. In this case, practitioners [8] take as a base for fees the corresponding notional $N^{**}_{i^{**}}$ such that the cost is rather $v_0^{**}N^{**}_{i^{**}}$ since the term $v_0^{**}H^{**}_{t,i^{**}}$ vanishes.

The hedging problem for the initial portfolio V is reduced to suitably choose the financial instruments with values

$$H^{**}_{.,1}.....H^{**}_{.,i^{**}}......H^{**}_{.,I^{**}} \text{ and } H^{*}_{.,1}.....H^{*}_{.,i^{*}}......H^{*}_{.,I^{*}}$$

and the corresponding security numbers

$$n_1^{**},...,n^{**}_{i^{**}}....,n^{**}_{I^{**}} \text{ and } n_1^{**},...,n^{**}_{i^{**}}....,n^{**}_{I^{**}}$$

such that the value of

$$\left|P\,\&\,L\_\text{cov}ered\_portfolio_{t,t+\delta}(.)\right|$$

should be small as possible. The difficulty here is linked to the fact that the future values of the hedging instruments at time $(t+\delta)$, are unknown at the present time t where the hedge strategy is built. The choice of the hedging instruments is dictated by the willing that the resultant effect of their change variations would roughly offset (i.e. going in the opposite direction) the change of the portfolio V to hedge. Then, the problem is reduced to a minimization problem of finding suitable allocation for the security numbers

$$n_1^{**},...,n^{**}_{i^{**}}....,n^{**}_{I^{**}},..\,n_1^{**},...,n^{**}_{i^{**}}....,n^{**}_{I^{**}}$$

Under PS or (5) the point is to assume that for any nonnegative integer p one has the approximation

$$\begin{aligned} &U_{t+\delta}(.)-U_t \approx Sens(0;t,\delta,U) \\ &+\sum_{k=1}^{p}\frac{(-1)^k}{k!}Sens(k;t,U)\varepsilon^k(.) \end{aligned} \tag{11}$$

where U is one of V , $H^{**}_{t,i^{**}}$ and $H^{*}_{t,i^{*}}$. In (11) the notations

$$Sens(0;t,\delta,V) \quad \text{and} \quad Sens(k;t,\delta,V)$$

are used to refer respectively the zero and k-th sensitivities order of the considered financial instrument V, computed at time t and are assumed to prevail for the horizon δ . A main point on the efficiency of (11) in the hedging operation relies on the suitable choice of the integer p such that the approximation-error

$$R(.)=\left|\begin{matrix} U_{t+\delta}(.) \\ -U_t-\left(\begin{matrix} Sens(0;t,\delta,U) \\ +\sum_{k=1}^{p}\frac{(-1)^k}{k!}Sens(k;t,\delta,U)\varepsilon^k(.)\end{matrix}\right)\end{matrix}\right| \tag{12}$$

is small from the perspective of the hedger, as for example $R(.) \le 10^{-2}$. Such a strong requirement may be useful since very often in practice one has to deal with positions having large notional size as nU_t with n = 107. Making use of (11) for $U=V$, $U=H^{**}$ and $U=H^{*}$ and taking (9) and (10) into account, then one has

$$\begin{aligned} &P\,\&\,L\_\text{cov}ered\_portfolio_{t,t+\delta}(.) \\ &\approx\left(\theta_0^V+\sum_{i^{**}=1}^{I^{**}}\theta^{**}_{0,i^{**}}n^{**}_{i^{**}}-\sum_{i^{*}=1}^{I^{*}}\theta^{*}_{0,i^{*}}n^{*}_{i^{*}}\right) \\ &+\sum_{k=1}^{p}\frac{(-1)^k}{k!}\left(\theta_k^V+\sum_{i^{**}=1}^{I^{**}}\theta^{**}_{k,i^{**}}n^{**}_{i^{**}}-\sum_{i^{*}=1}^{I^{*}}\theta^{*}_{k,i^{*}}n^{*}_{i^{*}}\right)\varepsilon^k(.) \end{aligned} \tag{13}$$

where

$$\theta_0^V \equiv Sens(0;t,\delta,V) \tag{14}$$

$$\begin{aligned} &\theta^{**}_{0,i^{**}}=Sens(0;t,\delta,H^{**}_{.,i^{**}}) \\ &-\left\{\frac{1}{P(t,t+\delta)}-1\right\}\{v_0^{**}+v^{**}|h^{**}_{t,i^{**}}|\}N^{**}_{t,i^{**}}n^{**}_{i^{**}} \end{aligned} \tag{15}$$

$$\theta^{*}_{0,i^*} = Sens(0;t,\delta,H^{*}_{.,i^*}) - \left\{\frac{1}{P(t,t+\delta)} - 1\right\}\left\{v^{*}_{0} + v^{*}\left|h^{*}_{t,i^*}\right|\right\} N^{*}_{t,i^*} n^{*}_{i^*} \quad (16)$$

$$\theta^{V}_{k} \equiv Sens(k;t,\delta,V),$$
$$\theta^{**}_{k,i^{**}} = Sens(k;t,\delta,H^{**}_{.,i^{**}}), \theta^{*}_{k,i^*} = Sens(k;t,\delta,H^{*}_{.,i^*}) \quad (17)$$

The idea for obtaining the sensitivity for a portfolio is that this last is a linear combination of various instruments. So we are reduced to compute the sensitivity for each instrument. Next the sensitivity for a given instrument (linear with respect to the zero-coupon bonds) depends just on the sensitivities of the involved zero-coupons. It means that the main point is roughly speaking to derive the sensitivity of a zero-coupon. Due to the space limitation, these sensitivities $Sens(k;t,\delta,V)$'s are not reported here but the full details may be seen over our complete technical paper [Ja-Ra-Ya; 2012].

We refer as a view on the interest rate shift $\varepsilon(.)$, the hypothesis that there are nonnegative real numbers $\varepsilon^{\bullet}$ and $\varepsilon^{\bullet\bullet}$ for which

$$-\varepsilon^{\bullet} \le \varepsilon \le \varepsilon^{\bullet\bullet} \quad (18)$$

Though $\varepsilon(.)$ is a random quantity, not known at the present time t, with historical data on zero-coupon prices, it is not hard for the practitioner to infer deterministic values of $\varepsilon^{\bullet}$ and $\varepsilon^{\bullet\bullet}$ corresponding to the available past prices. But she can also incorporate her view for the situation at the considered future horizon δ. Starting from (13), using the view (18), and neglecting the remainder term then the quantity $\left|P\&L\_covered\_portfolio_{t,t+\delta}(.)\right|$ is essentially bounded by

$$F(n^{**}_{1},\dots,n^{**}_{i^{**}}\dots,n^{**}_{I^{**}},n^{*}_{1},\dots,n^{*}_{i^*}\dots,n^{*}_{I^{**}},\varepsilon^{\bullet},\varepsilon^{\bullet\bullet}) = \left(\theta^{V}_{0} + \sum_{i^{**}=1}^{I^{**}}\theta^{**}_{0,i^{**}}n^{**}_{i^{**}} - \sum_{i^{*}=1}^{I^{*}}\theta^{*}_{0,i^*}n^{*}_{i^*}\right) + \sum_{k=1}^{p}\frac{(-1)^k}{k!}\left(\theta^{V}_{k} + \sum_{i^{**}=1}^{I^{**}}\theta^{**}_{k,i^{**}}n^{**}_{i^{**}} - \sum_{i^{*}=1}^{I^{*}}\theta^{*}_{k,i^*}n^{*}_{i^*}\right)\varepsilon^{k}(.) \quad (19)$$

This last may be seen as the objective function associated with a minimization problem and related to the hedging issue presented above. All remainder terms may be removed after choosing the expansion order p sufficiently large. For the function F as defined in (19), we are lead to an integer optimization problem defined by integer linear constraints, since the objective function is both non-linear, non-convex and non-differentiable at the origin. To overcome these difficulties we make use of a linearization technique as introduced in [9] and which7 consists to replace the initial problem by an equivalent linear problem. However at last, a solver as the commercial CPLEX solver 9.0 is useful to solve the resulting Mixed Integer Linear Problem we introduce. More details are given in [1].

III. Numerical Illustration

The present time-t shape of the yield curve may be seen as interpolated from available market interest rates by using the Nelson-Siegel-Svenson model [10] as

$$y(t;\tau) = \beta_{t;1} + \beta_{t;2}b_2(\tau\gamma) + \beta_{t;3}b_3(\tau\gamma) \quad (20)$$

with

$$b_2(u) = \frac{1-\exp(-u)}{u} \text{ and } b_3(\tau\gamma) = b_2(u) - \exp(-u)$$

here $\beta_{t;1}, \beta_{t;2}, \beta_{t;3}$ and γ depend on time t but not on the time-to-maturity τ. The model is assumed to be calibrated as

$$\beta_{t;1} = 0.0758; \beta_{t;2} = -0.02098; \beta_{t;3} = -0.00162; \gamma = 0.609$$

We consider a hedging horizon of $\delta = 90 days$ To take into consideration hedging costs, the deposit rates linked to holding the position (either for payer or receiver swap) are assumed to be given by $v^{**}_0 = v^{*}_0 = 20\%$. For the time-horizon $t+\delta$ the interest rate is supposed to make a PS with respect to the view $-\varepsilon^{\bullet} = -3\%$ and $\varepsilon^{\bullet\bullet} = 3\%$ .The choice $p = 12$ is chosen here in order to insure remainder terms with small sizes, which consequently can be neglected. The notations with tilde ($\tilde{}$) are used in the sequel to refer the portfolio to hedge. We are interested here to hedge a swap portfolio by another swap portfolio. The portfolio to cover is assumed to be made by five types of payer swaps $\tilde{S}^{**}_1$ to $\tilde{S}^{**}_5$ and three types of $\tilde{S}^{*}_1$ to $\tilde{S}^{*}_3$. The characteristics of these swaps are summarized in Table I.

TABLE I: Characteristics of the Potfolio to Hedge

type	number	maturity	frequency	rate_Swap
S^{**}_1	100	3years	6months	6.65%
$\tilde{S}^{**}_2$	200	4years	6year	6.82%
$\tilde{S}^{**}_3$	300	7years	6year	7.11%
$\tilde{S}^{**}_4$	100	10years	6months	7.25%
$\tilde{S}^{**}_5$	200	5years	6months	6.94%
$\tilde{S}^{*}_1$	200	4years	1year	6.93%
$\tilde{S}^{*}_2$	100	6years	1year	7.17%
$\tilde{S}^{*}_3$	100	7years	1year	7.24%

Names of the types of swaps used are displayed in the first column of this Table I. The numbers of swaps used for each type are presented in the second column. Maturities of the considered swaps are given in the third column. We have written in the fourth column the corresponding swap payment frequency, as semi-annually or annually frequency-based. Each swap is assumed to have the notional value of 1 Million Euro. The fair rate swap of each swap is given in the fifth column. Assuming that the present time corresponds to the time-inceptions for all of these swaps, and then the portfolio under consideration has zero value.

In order to decide to hedge or not the considered swap portfolio, it is valuable to have a projection of the low and high bounds for the portfolio change value at the given horizon and under the view of (more and less severe) parallel shift mentioned above. When applying a criterion we introduce in our full paper [1], then one obtain the result which is summarized in Table II.

TABLE II: BOUND OF THE PORTFOLIO TO COVER

Channels	$\in^*$	$change\_value\_portf_{t,t+\delta}(\in^*)$
$change\_value\_Swap_{t,t+\delta}(.)_{\min}$	-3%	-2.36*10^7
$change\_value\_Swap_{t,t+\delta}(.)_{\max}$	3%	-1.93*10^7

In Table II, by $\in^*$ we denote the value of $\varepsilon \in [-3\%;3\%]$ which allows to attain the minimum or maximum of the portfolio change value. It is seen here that in the worst case, the potential loss when dealing with just a naked portfolio position can attain the size of 20 Millions of Euros, which corresponds roughly to 20 swaps. So it might be useful to hedge the position as we consider now.

To hedge the previous portfolio introduced in Table II above, we make use of another swap portfolio made by one type of payer swap S_1^{**}, and three types of receiver swaps S_1^*, S_2^*, S_3^* The characteristics of all of these instruments are summarized in Table III.

TABLE III: CHARACTERISTICS OF THE HEDGING INSTRUMENTS

type	number	maturity	frequency	rate_Swap
S_1^{**}	n_1^{**}	2years	6months	6.41%
S_1^*	n_1^*	3years	1year	6.76%
S_2^*	n_2^*	10years	1year	7.37%
S_3^*	n_3^*	8years	6months	7.17%

The amount required for the hedging depends actually on the number of instruments used for that purpose, and consequently is not known in advance. As detailed in [1] here we can take D = 65 Million Euros. The numbers n_1^{**} and n_1^* , n_2^* , n_3^* of swaps S_1^{**} and S_1^*, S_2^*, S_3^* 3respectively required for the hedging, obtained from the approach introduced in this work are finally summarized in Table IV.

TABLE IV: RESULT OF THE HEDGING

n_1^{**}	n_1^*	n_2^*	n_3^*	Max Profit Loss
0	122	0	84	883 737.24

The real Profits or Losses (PL) corresponding to some shifts $\varepsilon \in [-3\%;3\%]$ are presented in the second column of Table V. So by PLport we mean the PL corresponding to the naked portfolio change value (that is the portfolio PL in absence of hedging

TABLE V: WEALTH FOR ANY SHIFT AFTER THE HEDGING.

$\in$	PLport	PLinst	PLport_cov	ret_port_cov	Ret_port
-3%	-23889286	23614943	-857471	-1.32%	-36.75%
-2.5%	-19631335	19388781	-825683	-1.27%	-30.20%
-2%	-15513978	15291407	-805700	-1.24%	-23.87%
-1.5%	-11530194	11318597	-794726	-1.22%	-17.74%
-1%	-7673324	7466276	-790176	-1.22%	-11.81%
-0.5%	-3937044	3730514	-789658	-1.21%	-6.06%
0%	-315358	107520	-790967	-1.22%	4.92%
0.5%	9917602	-10120810	-786336	-1.21%	15.26%
1%	6606704	-6814674	-791098	-1.22%	10.16%
1.5%	9917602	-10120810	-786336	-1.21%	15.26%
2%	13134977	-13328061	-776212	-1.19%	20.21%
2.5%	16263436	-16439598	-759291	-1.17%	25.02%
3%	19307348	-19458483	-734263	-1.13%	29.70%

Profits and losses for the hedging instruments, denoted here PLinst and defined in (8) are displayed in the third column. In the fourth column one can see the PL for the overall portfolio (portfolio to hedge and hedging portfolio). These last quantities include the hedging costs as defined in (9). By ret_port, in the fifth column, we mean the ratio

$$ret\_port\_\mathrm{cov} = \frac{PLport\_\mathrm{cov}}{D}$$

It may be noted that it is not the return linked to the covered portfolio as we just take as a basis the maximal amount allowed for the hedging operation. Indeed for swaps whose the initial values may be equal to zero, the notion of return should be taken with care as it is analyzed by A. Meucci [8]. Observe that the portfolio to hedge is not assumed to be unwound at the considered horizon, and the amount D is freezed for the hedge though the cost really involved in the operation is strictly less than D. For the last sixth column by ret_port we mean the ratio

$$ret\_port = \frac{PLport}{D}$$

The compensation between the loss related to the portfolio to hedge and the gain associated with the hedging portfolio may be understood from the alternated signs for the quantities displayed in the second and third columns. For $\varepsilon = 0\%$ one has $ret\_port = 049\%$ This an indication that the time-passage matters in hedging, and consequently should be taken into account as is the case for the sensitivities we have introduced in this paper. For the interest rate shift $\varepsilon = -2\%$ it may be seen, from the last two columns, that $ret\_port\_\mathrm{cov} = -1.24\%$ and $ret\_port = -23.87\%$. This means that a loss appears though the portfolio position is hedged or not. However the magnitude is clearly more important than the one involved in absence of hedge. Under the shift $\varepsilon = 2\%$ one has $ret\_port\_\mathrm{cov} = -1.19\%$ and $ret\_port = -20.21\%$ That is, in absence of the hedging operation, the considered portfolio has lead to an important gain. The hedge has an effect to get at worst a loss, but the corresponding magnitude (when taking into account D as a reference basis) is fortunately small. The cost of the hedging instruments is about 583128.94. Here the resulting loss can be viewed as the price of uncertainty and fear about the interest rate behavior at the considered horizon. At this point, it may be important to recall that the hedging operation has mainly as purpose to roughly maintain the portfolio at its initial level, but not to make any profit.

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