Parametric Control of Indicators Volatility of the National Economy of Kazakhstan within the Framework of Regional Economic Union

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***Abstract*—A dynamic stochastic general equilibrium model of Kazakhstan, Belarus and Russia, forming the Customs Union, and the rest of the world has been developed. The estimation of the model parameters has been done on the basis of statistics of Kazakhstan, Belarus, Russia and the rest of the world (in the case of the Organization for Economic Cooperation and Development). Macroeconomic analysis is conducted for effects of shocks on the economic performance of the national economy of Kazakhstan. The problem of minimizing the volatility of macroeconomic indicators of the national economy of Kazakhstan (GDP and inflation) based on the approach of the theory of parametric control has been formulated and solved.**

***Index Terms*—National economy, regional economic union, DSGE model, macroeconomic indicators volatility, parametric control.**

I. Introduction

There is a tendency around the world of creation of regional economic unions. The main reason for this trend is the desire to improve the economic performance of both national economies and the economy of the entire region. The European Union, the North American Free Trade Agreement, the Asia-Pacific Economic Cooperation and etc. can serve as examples.

Kazakhstan, Belarus and Russia established the Customs Union in 2010 with a view to its subsequent transformation into the Common Economic Space.

The countries in the regional unions are facing with a problem of carrying out the economic policy within the framework of these unions, i.e. taking into account the country's relationship with the countries of the union and the rest of the world. The dynamic stochastic general equilibrium models (DSGE models) [1] are recently becoming a widespread tool of macroeconomic analysis in the framework of regional unions. One of the important issues of macroeconomic analysis is assessment of the impact of internal and external shocks on the performance of national economies.

The well-known literature describes the regional unions within the framework of the DSGE models in the following way:

1) Representation of the regional union as one country and the rest of the world as another country [2], [3];
2) Representation of the regional union as one country and the rest of the world as a number of countries [4];
3) Representation of the regional union as two countries – the country under study and the rest of the regional union, and excluding the rest of the world [5]-[7];
4) Representation of the regional union as two countries – the country under study and the rest of the regional union, and the rest of the world as a number of countries [8].

This paper provides DSGE model of the Customs Union and the rest of the world (hereinafter the Model), which differs from the known representations of regional unions that considered regional union is presented in terms of three separate countries, interacting both with each other and with the rest of the world (presented in the form of one country). Relationship between the economies of the countries in the Model is through the flows of goods and capital.

The main differences of the Model from well-known DSGE models are also:

1) Collection of customs duties in trade of the Customs Union with the rest of the world and the distribution of collected duties between three countries of the Customs Union;
2) Representation of the government budget expenditures of the Customs Union member countries using appropriate regression functions of GDP and government debt;
3) Representation of the government budget revenues of the Customs Union member countries as the sum of the collected taxes, duties and oil incomes;
4) Description of the government budget deficits of the Customs Union member countries;
5) Introduction of the oil sector.

The paper presents the setting of the problem of parametric control [9] of macroeconomic indicators volatility based on the Model and results of its solving.

II. Description of the Model

Within the framework of the developed Model, each country has the household sector, the goods production sector, the oil sector and the government sector, which interact in a stochastic environment. The activities of relevant sectors in the countries of the Customs Union are equal and the activities of sectors of the rest of the world are simplified.

The activities of households consist in maximizing of the expected discounted sum of utilities, which depend on the consumption and labor. Based on the solution of this optimization problem, the households purchase consumer and investment goods, government bonds and provide capital

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for lease. Composition of consumer and investment goods is determined in terms of cost minimization. It is assumed that the labor market is monopolistically competitive, and households set the wages, taking into account the Calvo stickiness [10].

Good producers hire the labor of households, rent capital and purchase oil to produce goods in terms of minimizing the spending to production factor. The goods market is assumed to be monopolistically competitive, and producers set the price for their goods taking into account the Calvo stickiness based on the maximization of expected discounted sum of cash flows.

The government sets the interest rate on government bonds, collect taxes and customs duties, borrow money from households and carry out government spendings.

In the Model, economic sectors affected by the following shocks: the preference shock, affecting the objective function of households; investment shock, reflected in the accumulation of capital; productivity shock, explaining the changes in production factor effectiveness; wages markup shock at setting of wages by the households; capital markup shock, introduced as a stochastic financial premium; price markup shock at setting the prices for goods; shock of risk premium that households have to pay when they borrow from abroad, government spending and interest rates shocks, affecting economic policy, oil production shock, oil price shock and etc. All shocks are given in the form of the first-order autoregressions or i.i.d. white noises.

The Model is formed on the basis of equations of the first-order conditions of the above optimization problems of household and goods production sectors, the rules of the economic activities of the oil sector, government and setting of shocks. The model has the following form in vector terms:

$$E_t F^{\theta}(X_{t-1} X_t X_{t+1,} H_t^{\Sigma_H}) = 0 \tag{1}$$

Here E_t is a sign of conditional mathematical expectation on information available at time t ($t = 1,2,\ldots$); F^{θ} is a known vector function; θ is a set of parameters consisting of the structural parameters of the model and autoregressive parameters of shocks; X_t is a vector consisting of the endogenous variables and shocks, set by the first-order autoregression; X_0 is initial value of vector X_t; $H_t^{\Sigma_H}$ is a vector consisting of i.i.d. white noises, Σ_H is a set of parameters consisting of the standard deviations of noises of $H_t^{\Sigma_H}$.

It is difficult to find an exact solution for the obtained nonlinear Model, therefore we use the common approach [11] of finding an approximate solution by log-linearization equations around steady state of the model (1). Using this approach allows to transform the nonlinear model into a linear model with rational expectations in the following view:

$$A^{\theta}\mathrm{E}_t\hat{X}_{t+1} + B^{\theta}\hat{X}_t + C^{\theta}\hat{X}_{t-1} + D^{\theta}H_t^{\Sigma_H} = 0 \tag{2}$$

where $A^{\theta}, B^{\theta}, C^{\theta}, D^{\theta}$ are the matrices of appropriate dimensions, $\hat{X}_t$ is a vector of endogenous variables of the linear model corresponding to X_t, the vector of endogenous variables of nonlinear model.

Below the linear DSGE model of the Customs Union and the rest of the world (2) is shown in expanded form. The designation of variables by " ^ " means the log-deviation in percentage of the relevant either economic indicators or autoregression shocks from their steady state values. Variables indexed by notation $"kz", "by", "ru", "rw"$ correspond to national economies of Kazakhstan, Belarus, Russia and the rest of the world economy, respectively.

The dynamics of the i country households consumption is given by the following equation (hereinafter $i \in \{kz, by, ru\}$, unless otherwise specified):

$$\hat{c}_t^i = \frac{h^i}{1+h^i}\hat{c}_{t-1}^i + \frac{1}{1+h^i}\mathrm{E}_t\hat{c}_{t+1}^i - \frac{1-h^i}{(1+h^i)\sigma_C^i}\Big(\hat{r}_t^i - \mathrm{E}_t\hat{\pi}_{t+1}^i + \mathrm{E}_t\hat{\varepsilon}_{C,t+1}^i - \hat{\varepsilon}_{C,t}^i - \frac{\tau_C}{1+\tau_C}\mathrm{E}_t\hat{\tau}_{C,t+1}^i + \frac{\tau_C}{1+\tau_C}\hat{\tau}_{C,t}^i\Big) \tag{3}$$

where c_t^i is households consumption; r_t^i is government bond yield; π_t^i is inflation; $\hat{\tau}_{C,t}^i$ is effective VAT rate; h^i ($0 < h^i < 1$) is the coefficient of consumption habits of households; σ_C^i ($\sigma_C^i > 0$) is inverse intertemporal elasticity of substitution of households consumption; τ_C is official VAT rate; $\varepsilon_{C,t}^i$ is preferences shock of households.

Equation of household investment of i country to j country is given as follows ($j \in \{kz, by, ru, rw\}$):

$$0 = \hat{q}_t^i + \hat{\varepsilon}_{X,t}^j - S^i\big(\hat{x}_t^{i,j} - \hat{x}_{t-1}^{i,j}\big) + S^i\beta\big(\mathrm{E}_t\hat{x}_{t+1}^{i,j} - \hat{x}_t^{i,j}\big) \tag{4}$$

Here $x_t^{i,j}$ is investment of household i country to j country; q_t^i is the capital value; β is the discount factor of utilities used by households within the objective function; $\varepsilon_{X,t}^i$ is investment shock; S^i is the second derivative at the steady state of the cost function at the accumulation of capital.

Equations defining optimal amount of capital of the country i in the country j specified in the following forms ($j \in \{kz, by, ru, rw\}$):

$$\hat{q}_t^i + \hat{p}_{X,t}^i = \mathrm{E}_t\Big(\hat{\varepsilon}_{C,t+1}^i - \hat{\varepsilon}_{C,t}^i + \frac{\tau_C^i}{1+\tau_C^i}\hat{\tau}_{C,t}^i - \frac{\tau_C^i}{1+\tau_C^i}\hat{\tau}_{C,t+1}^i - \sigma_C\frac{\hat{c}_{t+1}^i - h^i\hat{c}_t^i}{1-h^i} + \sigma_C\frac{\hat{c}_t^i - h^i\hat{c}_{t-1}^i}{1-h^i} + \beta(1-\tau_K^i)R_k^i\Big(-\frac{\tau_K^i}{1-\tau_K^i}\hat{\tau}_{K,t+1}^i + \hat{r}_{k,t+1}^i\Big) + \beta\tau_K^i\delta\big(\hat{\tau}_{K,t+1}^i + \hat{p}_{X,t+1}^i\big) + \beta(1-\delta)(\hat{q}_{t+1}^i + \hat{p}_{X,t+1}^i)\Big) + u_{Q,t}^i \tag{5}$$

$$\hat{q}_t^i + \hat{p}_{X,t}^i = \mathrm{E}_t\Big(\hat{\varepsilon}_{C,t+1}^i - \hat{\varepsilon}_{C,t}^i + \frac{\tau_C^i}{1+\tau_C^i}\hat{\tau}_{C,t}^i - \frac{\tau_C^i}{1+\tau_C^i}\hat{\tau}_{C,t+1}^i - \sigma_C\frac{\hat{c}_{t+1}^i - h^i\hat{c}_t^i}{1-h^i} + \sigma_C\frac{\hat{c}_t^i - h^i\hat{c}_{t-1}^i}{1-h^i} + \beta(1-\tau_K^j)R_k^j\Big(-\frac{\tau_K^j}{1-\tau_K^j}\hat{\tau}_{K,t+1}^j + \hat{r}_{k,t+1}^j + \hat{k}_{t+1}^{i,j}\Big) + \beta\tau_K^j\delta\big(\hat{\tau}_{K,t+1}^j + \hat{p}_{X,t+1}^j\big) + \beta(1-\delta)\mathrm{E}_t(\hat{q}_{t+1}^i + \hat{p}_{X,t+1}^j)\Big) + u_{Q,t}^i \tag{6}$$

Here $p_{X,t}^i$ is investment price index; $r_{K,t}^i$ is rental rate of capital; $\hat{\tau}_{K,t}^i$ is effective tax rate of capital income; δ is depreciation rate of capital; $k_t^{i,j}$ is the accumulated amount of capital of the country i in the country j; τ_K^i is official tax rate of capital income; R_k^i is steady state of rental rate of capital; $\hat{u}_{Q,t}^i$ is capital markup shock.

The equation specifying the amount of accumulated capital is determined according to the formula ($j \in \{kz, by, ru, rw\}$):

$$\hat{k}_t^{i,j} = (1 - \delta)\hat{k}_{t-1}^{i,j} + \delta\hat{\varepsilon}_{X,t}^j + \delta\hat{x}_t^{i,j} \quad (7)$$

Equation for inflation of domestic goods prices in the country:

$$\hat{\pi}_t^{i,i} = \frac{(1-\xi_P^i)(1-\beta\xi_P^i)}{\xi_P^i}\left(\widehat{mc}_t^i - \hat{p}_t^{i,i} + u_{\mu_p,t}^i\right) + \beta \mathrm{E}_t\hat{\pi}_{t+1}^{i,i} - \beta\gamma_P^i\hat{\pi}_t^{i,i} + \gamma_P^i\hat{\pi}_{t-1}^{i,i} \quad (8)$$

where

$$\hat{\pi}_t^{i,i} = \hat{p}_t^{i,i} - \hat{p}_{t-1}^{i,i} + \hat{\pi}_t^i \quad (9)$$

Here $p_t^{i,i}$, $\pi_t^{i,i}$ are the price level and inflation of prices of domestic goods, respectively; mc_t^i is the marginal cost of production; ξ_P^i $(0 < \xi_P^i < 1)$, $\gamma_P^i(0 < \gamma_P^i < 1)$ are the probability of non-optimal setting of prices by producer and the degree of indexation of prices according to the Calvo pricing model, respectively; $u_{\mu_p,t}^i$ is price markup shock.

The equation of real wages is given in the following form:

$$\hat{w}_t^i(1+\beta) = \beta \mathrm{E}_t\hat{w}_{t+1}^i + \hat{w}_{t-1}^i + \beta \mathrm{E}_t\hat{\pi}_{t+1}^i - (1+\beta\gamma_W^i)\hat{\pi}_t^i + \gamma_W^i\hat{\pi}_{t-1}^i + \frac{(1-\beta\xi_W^i)(1-\xi_W^i)}{\xi_W^i\left(1+\frac{(1+\lambda_W^i)\sigma_L^i}{\lambda_W^i}\right)}\left(\sigma_L^i\hat{l}_t^i + \hat{\varepsilon}_{L,t}^i + \frac{\sigma_C^i}{1-H^i}\left(\hat{c}_t^i - h^i\hat{c}_{t-1}^i\right) - \hat{w}_t^i + \frac{\tau_W^i}{1-\tau_W^i}\hat{\tau}_{W,t}^i + \frac{\tau_C^i}{1+\tau_C^i}\hat{\tau}_{C,t}^i + u_{\mu_w,t}^i\right) \quad (10)$$

Here w_t^i is the level of real wages of households; l_t^i is labor; ξ_W^i $(0 < \xi_W^i < 1)$, $\gamma_W^i(0 < \gamma_W^i < 1)$ are the probability of non-optimal setting of wages by the households and the degree of indexation of wages according to the Calvo pricing model, respectively; σ_L^i $(\sigma_L^i > 0)$ is the inverse elasticity of labor supply; λ_W^i $(\lambda_W^i > 1)$ is wage markup; $\varepsilon_{L,t}^i$ is the labor supply shock; $u_{\mu_w,t}^i$ is the wage markup shock; $\hat{\tau}_{W,t}^i$ is effective tax rate on wage; τ_W^i is official tax rate on wage.

The production function of the goods producers is given in accordance with the formula:

$$\hat{y}_{H,t}^i = \hat{\varepsilon}_{a,t}^i + \alpha_k^i\hat{k}_{t-1}^i + \alpha_L^i\hat{l}_t^i + (1-\alpha_k^i-\alpha_L^i)\hat{o}_{h,t}^i \quad (11)$$

Here $\hat{y}_{H,t}^i$ is output of non-oil goods; $\hat{k}_{t-1}^i$ is amount of utilized capital; $\hat{o}_{h,t}^i$ is amount of utilized oil; α_k^i, α_L^i $(0 < \alpha_k^i, \alpha_L^i < 1)$ are the output elasticities of capital and labor, respectively; $\hat{\varepsilon}_{a,t}^i$ is productivity shock.

The total capital represents as the sum of capital from each country:

$$\hat{k}_{t-1}^i = \sum_{j\in\{kz,by,ru,rw\}}\hat{k}_{t-1}^{j,i} \quad (12)$$

The equation, expressing relation between the labor and capital, is as follows:

$$\frac{\tau_S^i}{1+\tau_S^i}\hat{\tau}_{S,t}^i + \hat{w}_t^i + \hat{l}_t^i = \hat{k}_{t-1}^i + \hat{r}_{K,t}^i \quad (13)$$

where $\hat{\tau}_{S,t}^i$ is effective social tax rate; τ_S^i is official social tax rate.

The equation, expressing relation between the labor and oil, is as follows:

$$\frac{\tau_S^i}{1+\tau_S^i}\hat{\tau}_{S,t}^i + \hat{w}_t^i + \hat{l}_t^i = \hat{o}_{h,t}^i + \hat{p}_{O,t}^i \quad (14)$$

where $\hat{p}_{O,t}^i$ is oil price.

Equation of the marginal cost can be presented as:

$$\widehat{mc}_t^i = \alpha_k^i\hat{r}_{K,t}^i + \alpha_L^i\left(\frac{\tau_S^i}{1+\tau_S^i}\hat{\tau}_{S,t}^i + \hat{w}_t^i\right) + (1-\alpha_k^i-\alpha_L^i)\hat{o}_{h,t}^i, -\hat{\varepsilon}_{a,t}^i \quad (15)$$

Employment equation is given as follows:

$$\Delta\hat{e}_t^i = \frac{(1-\xi_E^i)(1-\beta\xi_E^i)}{\xi_E^i}\left(l_t^i - \hat{e}_t^i\right) + \beta \mathrm{E}_t\Delta\hat{e}_{t+1}^i \quad (16)$$

where

$$\Delta\hat{e}_t^i = \hat{e}_t^i - \hat{e}_{t-1}^i \quad (17)$$

Here $\hat{e}_t^i$ is employment; ξ_E^i $(0 < \xi_E^i < 1)$ is the probability of non-optimal setting of employment according to the Calvo model.

The equation describing uncovered interest parity condition is ($j \in \{kz, by, ru, rw\}\backslash i$):

$$\hat{r}_t^i = \hat{r}_t^j + \varrho^{i,j}\hat{b}_t^{i,j} + \mathrm{E}_t\left(\hat{s}_{t+1}^{i,j} - \hat{s}_t^{i,j}\right) + \hat{\varepsilon}_{pr,t}^i \quad (18)$$

Here $\hat{s}_t^{i,j}$ is the nominal exchange rate; $\hat{\varepsilon}_{pr,t}^i$ is risk premium shock; $\varrho^{i,j}$ is elasticity of risk premium.

Equation specifying the relationship between the real and nominal exchange rates is ($j \in \{kz, by, ru, rw\}\backslash i$):

$$\widehat{rer}_t^{i,j} - \widehat{rer}_{t-1}^{i,j} = \hat{s}_t^{i,j} + \hat{s}_{t-1}^{i,j} - \hat{\pi}_t^i + \hat{\pi}_t^j \quad (19)$$

where $\widehat{rer}_t^{i,j}$ is real exchange rate.

Oil production and oil price are specified as autoregressions:

$$\hat{y}_{o,t}^i = \rho_{Y_o}^i\hat{y}_{o,t}^i + u_{Y_o,t}^i \quad (20)$$

$$\hat{p}_{o,t}^{rw} = \rho_{P_o}^{rw}\hat{p}_{o,t}^{rw} + u_{P_o,t}^{rw} \quad (21)$$

where $\rho_{Y_o}^i, \rho_{P_o}$ are autoregressive coefficients; $u_{Y_o,t}^i, u_{P_o,t}^i$ are shocks of oil production and oil price.

Oil price in national currency:

$$\hat{p}_{o,t}^i = \widehat{rer}_t^{rw,i}\hat{p}_{o,t}^{rw} \quad (22)$$

where $\hat{p}_{o,t}^{rw}$ is world oil price.

Equation for GDP is defined as follows:

$$\hat{y}_t^i = \frac{Y_h^i}{Y^i}\hat{y}_{h,t}^i + \frac{Y_O^i}{Y^i}\hat{y}_{O,t}^i \quad (23)$$

where $\hat{y}_t^i$ is GDP; Y_h^i and Y_O^i are steady states of non-oil and oil outputs, respectively.

Taylor rule [12] determining the interest rate on government borrowing is given as follows:

$$\hat{r}_t^i = \rho_r^i \hat{r}_{t-1}^i + (1-\rho_r^i)\{r_{r\pi}^i \hat{\pi}_t^i + r_{ry}^i \hat{y}_t^i + r_{rrer}^i \widehat{rer}_{rw,t}^i\} + r_{rd\pi}^i(\hat{\pi}_t^i - \hat{\pi}_{t-1}^i) + r_{rdy}^i(\hat{y}_t^i - \hat{y}_{t-1}^i) + r_{rdrer}^i(\widehat{rer}_{rw,t}^i - \widehat{rer}_{rw,t-1}^i) + u_{r,t}^i \quad (24)$$

Here ρ_r^i is smoothing factors; $r_{r\pi}^i, r_{ry}^i, r_{rrer}^i, r_{rd\pi}^i, r_{rdy}^i$, r_{rdrer}^i are weight coefficients; $u_{r,t}^i$ is shock of interest rates.

The rule of government spendings is set as follows:

$$\widehat{ge}_t^i = \rho_{ge}^i \widehat{ge}_{t-1}^i - (1-\rho_{ge}^i)(r_{geb}^i \hat{b}_t^i + r_{gey}^i \hat{y}_t^i) + u_{ge,t}^i \quad (25)$$

Here $\widehat{ge}_t^i$ is the amount of government spendings; $\hat{b}_t^i$ is total government debt; ρ_{ge}^i is smoothing factor; r_{geb}^i, r_{gey}^i are weight coefficients; $u_{ge,t}^i$ is shock of government spendings.

The ratio between the government (budget) spendings and government consumption is given as follows:

$$\widehat{ge}_t^i = \hat{g}_t^i + \hat{p}_{G,t}^i - \hat{\varepsilon}_{g,t}^i \quad (26)$$

Here $\hat{g}_t^i$ is government consumption; $p_{G,t}^i$ is the price index of government spendings; $\hat{\varepsilon}_{g,t}^i$ is shock of share of government consumption in government spendings.

The total government debt represents as the sum of government debt to each country:

$$\hat{b}_t^i = \sum_{j \in \{kz,by,ru,rw\}\backslash i} \frac{rer^{j,i}B^{i,j}}{B^i}(\hat{b}_t^{i,j} + \widehat{rer}_t^{j,i}) \quad (27)$$

Here, $\hat{b}_t^{i,j}$ is the government debt of country i to country j; $Rer^{j,i}$, $B^{i,j}$, B^i are steady state values of real exchange rate, government debt to j country and total government debt, respectively.

Government revenues are determined by the following equation (for $i \in \{kz, ru\}$):

$$\widehat{gr}_t^i = \chi^i \frac{Y_O^i P_O^i}{Gr^i}(\hat{y}_{O,t}^i + \hat{p}_{O,t}^i) + \frac{Y_O^i \tau_O^i}{Gr^i}(\hat{y}_{O,t}^i - \hat{c}_{O,t}^i - \hat{o}_{H,t}^i + \hat{\tau}_{O,t}^i) + \frac{C^i\tau_C^i}{Gr^i}(\hat{c}_t^i + \hat{\tau}_{C,t}^i) + \frac{(\tau_W^i+\tau_S^i)W^iL^i}{Gr^i}\left(\frac{\tau_W^i}{\tau_W^i+\tau_S^i}\hat{\tau}_{W,t}^i + \frac{\tau_S^i}{\tau_W^i+\tau_S^i}\hat{\tau}_{S,t}^i + \hat{w}_t^i + \hat{l}_t^i\right) + \frac{\tau_K^i R_K^i K^i}{Gr^i}(\hat{\tau}_{K,t}^i + \hat{k}_{t-1}^i + \hat{r}_{K,t}^i) - \frac{\tau_K^i \delta K^i P_{X,t}^i}{Gr^i}(\hat{\tau}_{K,t}^i + \hat{k}_{t-1}^i + \hat{p}_{X,t}^i) + \frac{Ct^i}{Gr^i}\widehat{ct}_t^i \quad (28)$$

(for $i = by$)

$$\widehat{gr}_t^i = \frac{C^i\tau_C^i}{Gr^i}(\hat{c}_t^i + \hat{\tau}_{C,t}^i) + \frac{(\tau_W^i+\tau_S^i)W^iL^i}{Gr^i}\left(\frac{\tau_W^i}{\tau_W^i+\tau_S^i}\hat{\tau}_{W,t}^i + \frac{\tau_S^i}{\tau_W^i+\tau_S^i}\hat{\tau}_{S,t}^i + \hat{w}_t^i + \hat{l}_t^i\right) + \frac{\tau_K^i R_K^i K^i}{Gr^i}(\hat{\tau}_{K,t}^i + \hat{k}_{t-1}^i + \hat{r}_{K,t}^i) - + \frac{\tau_K^i \delta K^i P_X^i}{Gr^i}(\hat{\tau}_{K,t}^i + \hat{k}_{t-1}^i + \hat{p}_{X,t}^i) + \frac{Ct^i}{Gr^i}\widehat{ct}_t^i \quad (29)$$

Here $\widehat{gr}_t^i$ is government revenues; $\widehat{ct}_t^i$ is customs duties; χ^i is the government-owned share of the oil sector; $\hat{\tau}_{O,t}^i$ is effective export duty rate on oil; τ_O^i is official export duty rate on oil; $Gr^i, Ct^i, K^i, P_X^i, W^i, L^i, C^i$ are steady states of government revenues, import customs duties, capital, investment price index, wage, labor and consumption, respectively.

Equation for import customs duties distributed to country i:

$$\widehat{ct}_t^i = \frac{N^i \tau_{rw,t} p^{rw,j}}{Ct^i}\sum_{k \in \{kz,by,ru\}}(C^{k,rw} + X^{k,rw} + G^{k,rw})\sum_{j \in \{kz,by,ru\}}\left(\frac{C^{j,rw}}{\sum_{k \in \{kz,by,ru\}}(C^{k,rw}+X^{k,rw}+G^{k,rw})}\left(\hat{c}_t^j - \varepsilon\left(\frac{\tau_{rw}}{1+\tau_{rw}}\hat{\tau}_{rw,t} + \hat{p}_t^{i,rw} - \hat{p}_t^j\right)\right) + \frac{X^{j,rw}}{\sum_{k \in \{kz,by,ru\}}(C^{k,rw}+X^{k,rw}+G^{k,rw})}\left(\hat{x}_t^j - \varepsilon\left(\frac{\tau_{rw}}{1+\tau_{rw}}\hat{\tau}_{rw,t} + \hat{p}_t^{i,rw} - \hat{p}_{X,t}^j\right)\right) + \frac{G^{j,rw}}{\sum_{k \in \{kz,by,ru\}}(C^{k,rw}+X^{k,rw}+G^{k,rw})}\left(\hat{g}_t^j - \varepsilon\left(\frac{\tau_{rw}}{1+\tau_{rw}}\hat{\tau}_{rw,t} + \hat{p}_t^{i,rw} - \hat{p}_{G,t}^j\right)\right) + \hat{\tau}_{rw,t} + \hat{p}_t^{rw,j}\right) \quad (30)$$

where $\hat{\tau}_{rw,t}$ is effective import duty rate; $\tau_{rw,t}$ is official import duty rate; $C^{k,rw}, X^{k,rw}, G^{k,rw}$ are steady states of export of goods to the rest of the world for consumption, investment and government spendings, respectively.

Equation of government budget deficit is set as follows:

$$\widehat{gds}_t^i = \frac{B^{i,i}}{Gds^i}\beta(\hat{b}_t^{i,i} - \hat{r}_t^i) - \frac{B^i}{Gds^i}(\hat{b}_{t-1}^{i,i} - \hat{\pi}_t^i) + \sum_{j \in \{kz,ru,by,rw\}\backslash i}\left(\frac{B^{i,j}}{Gds^i}\beta(\widehat{rer}_{i,t}^{j,i} + \hat{b}_{i,t}^{i,j} - \hat{r}_t^i) - \frac{B^{i,j}}{Gds^i}(\widehat{rer}_{i,t}^{j,i} + \hat{b}_{i,t-1}^{i,j} - \hat{\pi}_t^i)\right) \quad (31)$$

where $\widehat{gds}_t^i$ is budget deficit; Gds^i is steady state of budget deficit.

Budget balance equation is:

$$\widehat{ge}_t^i = \frac{Gr^i}{Ge^i}\widehat{gr}_t^i + \frac{Gds^i}{Ge^i}\widehat{gds}_t^i \quad (32)$$

Equations of government debt to other countries of the Customs Union and the rest of the world are:

$$\hat{b}_t^{i,j} = 0, (j \in \{kz, by, ru\}\backslash i) \quad (33)$$

$$\hat{b}_t^{i,rw} = \rho_b^{i,rw}\hat{b}_{t-1}^{i,rw} + u_{b,t}^{i,rw} \quad (34)$$

Here $\hat{b}^i_{rw,t}$ is government debt to the rest of the world; $\hat{b}^i_{j,t}$ is government debt to the country j of Customs Union; ρ^i_b is autoregressive coefficient; $u^i_{b,t}$ is shock of the government debt to the rest of the world.

Equations for tax rates:

$$\hat{\tau}^i_{C,t} = \rho^i_{\tau_C}\hat{\tau}^i_{C,t-1} + u^i_{\tau_C,t} \quad (35)$$

$$\hat{\tau}^i_{K,t} = \rho^i_{\tau_K}\hat{\tau}^i_{K,t-1} + u^i_{\tau_K,t} \quad (36)$$

$$\hat{\tau}^i_{S,t} = \rho^i_{\tau_S}\hat{\tau}^i_{S,t-1} + u^i_{\tau_S,t} \quad (37)$$

$$\hat{\tau}^i_{W,t} = \rho^i_{\tau_W}\hat{\tau}^i_{W,t-1} + u^i_{\tau_W,t} \quad (38)$$

$$\hat{\tau}^i_{rw,t} = \rho^i_{\tau_{rw}}\hat{\tau}^i_{rw,t-1} + u^i_{\tau_{rw},t} \quad (39)$$

$$\hat{\tau}^i_{O,t} = \rho^i_{O}\hat{\tau}^i_{O,t-1} + u^i_{O,t} \quad (40)$$

Here $\rho^i_{\tau_C}, \rho^i_{\tau_K}, \rho^i_{\tau_S}, \rho^i_{\tau_W}, \rho^i_{\tau_{rw}}, \rho^i_O$ are autoregressive coefficients; $u^i_{\tau_C,t}, u^i_{\tau_K,t}, u^i_{\tau_S,t}, u^i_{\tau_W,t}, u^i_{\tau_{rw},t}, u^i_{O,t}$ are i.i.d. white noises.

Equation for demand of goods of the country:

$$\hat{y}^i_{H,t} = \sum_{j\in\{kz,by,ru\}}\left(\frac{N^j}{N^i}P^{i,j^{-\varepsilon^i}}\gamma_C^{i,j}\frac{C^j}{Y^i}\left(\hat{c}^j_t - \omega^i\hat{p}^j_{O,t} - \varepsilon^i\hat{p}^{i,j}_{Z,t}\right) + \frac{N^j}{N^i}\left(\frac{P^{i,j}}{P^j_X}\right)^{-\varepsilon}\gamma_X^{i,j}\frac{X^j}{Y^i}\left(\hat{x}^j_t + \varepsilon^i\hat{p}^j_{X,t} - \varepsilon^i\hat{p}^{i,j}_t\right)\frac{N^j}{N^i}\left(\frac{P^{i,j}}{P^j_G}\right)^{-\varepsilon^i}\gamma_X^{i,j}\frac{G^j}{Y^i}\left(\hat{g}^j_t + \varepsilon^i\hat{p}^j_{g,t} - \varepsilon^i\hat{p}^{i,j}_t\right)\right) + P^{i,rw^{-\varepsilon}}\frac{Y^{rw}}{Y^i}\left(\hat{y}^{rw}_t - \varepsilon^i\hat{p}^{i,rw}_t\right) \quad (41)$$

Here $\frac{C^j}{Y^i}, \frac{X^j}{Y^i}, \frac{G^j}{Y^i}$ are the steady state values of the ratios of consumption, investments and government spendings of country j to GDP of country i; $P^{i,j}$ is the steady state values of prices in country j for goods of the country i; P^i_X, P^i_G are the steady state values of the price index of investments and government spendings, respectively; N^i share of GDP of Kazakhstan, Russia and Belarus in the total GDP of the Customs Union ($\sum_{j\in\{kz,by,ru\}}N^j = 1$), respectively; $\gamma_C^{i,j}, \gamma_X^{i,j}, \gamma_G^{i,j}$ are weight ratios in the formula of Dixit-Stiglitz on aggregation in consumption, investments and government spendings, respectively; ε^i is elasticity of substitution between the domestic and foreign goods; ω^i is elasticity of substitution between the goods and oil; $\hat{p}^{i,j}_{Z,t}$ is consumer price index excluding oil.

Equation for indices of consumer prices, prices of investment goods and government spendings:

$$0 = \gamma^i_C\hat{p}^i_{Z,t} + \gamma^i_O\hat{p}^i_{O,t} \quad (42)$$

$$\hat{p}^i_{Z,t} = \sum_{j\in\{kz,by,ru,rw\}}\gamma_C^{j,i}\hat{p}^{j,i}_t \quad (43)$$

$$\hat{p}^i_{X,t} = \sum_{j\in\{kz,by,ru,rw\}}\gamma_X^{j,i}\hat{p}^{j,i}_t \quad (44)$$

$$\hat{p}^i_{G,t} = \sum_{j\in\{kz,by,ru,rw\}}\gamma_G^{j,i}\hat{p}^{j,i}_t \quad (45)$$

Equation characterizing the law of one price ($j \in \{kz, by, ru, rw\}$; $i \neq j$):

$$\hat{p}^{i,i}_t = \widehat{rer}^{j,i}_t + \hat{p}^{i,j}_t \quad (46)$$

Rules for specifying shocks:

$$\hat{\varepsilon}^i_{a,t} = \rho^i_{\varepsilon_a}\hat{\varepsilon}^i_{a,t-1} + u^i_{\varepsilon_a,t} \quad (47)$$

$$\hat{\varepsilon}^i_{C,t} = \rho^i_{\varepsilon_C}\hat{\varepsilon}^i_{C,t-1} + u^i_{\varepsilon_C,t} \quad (48)$$

$$\hat{\varepsilon}^i_{L,t} = \rho^i_{\varepsilon_L}\hat{\varepsilon}^i_{C,t-1} + u^i_{\varepsilon_L,t} \quad (49)$$

$$\hat{\varepsilon}^i_{X,t} = \rho^i_{\varepsilon_x}\hat{\varepsilon}^i_{X,t-1} + u^i_{\varepsilon_x,t} \quad (50)$$

$$\hat{\varepsilon}^i_{g,t} = \rho^i_{\varepsilon_g}\hat{\varepsilon}^i_{g,t-1} + u^i_{\varepsilon_g,t} \quad (51)$$

$$\hat{\varepsilon}^i_{pr,t} = \rho^i_{\varepsilon_{Pr}}\hat{\varepsilon}^i_{pr,t-1} + u^i_{\varepsilon_{Pr},t} \quad (52)$$

where $\rho^i_{\varepsilon_a}, \rho^i_{\varepsilon_C}, \rho^i_{\varepsilon_L}, \rho^i_{\varepsilon_x}, \rho^i_{\varepsilon_g}, \rho^i_{\varepsilon_{Pr}}$ are autoregressive coefficients; $u^i_{\varepsilon_a,t}, u^i_{g,t}, u^i_{\varepsilon_C,t}, u^i_{\varepsilon_L,t}, u^i_{\varepsilon_x,t}$ are i.i.d. white noises. The remaining $u^i_{\mu_p,t}, u^i_{\mu_w,t}, u^i_{Q,t}, u^i_{r,t}, u^i_{ge,t}, u^i_{b,t}, u^i_{Y_o,t}, u^i_{P_o,t}, u^i_{\varepsilon_{Pr},t}$ shocks are presented as i.i.d. white noises.

We will capture the equations describing the economy of the rest of the world. They are obtained similarly to the corresponding equations of the Customs Union with the following simplifications: 1) the economy of the rest of the world is assumed to be closed, i.e. exports to the Customs Union member-countries and imports from these countries are considered as negligible in comparison with GDP of the rest of the world; 2) Consumption and investment present a fixed share of GDP of the rest of the world; 3) Setting of wages is flexible.

The obtained system of equations is similar to the standard New Keynesian model of three equations of GDP, inflation and interest rate [13]:

$$\hat{y}^{rw}_t = \frac{h^{rw}}{1+h^{rw}}\hat{y}^{rw}_{t-1} + \frac{1}{1+h^{rw}}\mathrm{E}_t\hat{y}^{rw}_{t+1} - \frac{1-h^{rw}}{(1+h^{rw})\sigma_C^{rw}}\left(\hat{r}^{rw}_t - \mathrm{E}_t\hat{\pi}^{rw,rw}_{t+1} + \hat{\varepsilon}^{rw}_{C,t+1} - \hat{\varepsilon}^{rw}_{C,t}\right) \quad (53)$$

$$\hat{\pi}^{rw,rw}_t = \frac{(1-\xi_P^{rw})(1-\beta\xi_P^{rw})}{\xi_P^{rw}}\left(\widehat{mc}^{rw}_t + u^{rw}_{\mu_p,t}\right) + \beta\mathrm{E}_t\hat{\pi}^{rw,rw}_{t+1} - \beta\gamma_P^{rw}\hat{\pi}^{rw,rw}_t + \gamma_P^{rw}\hat{\pi}^{rw,rw}_{t-1} \quad (54)$$

$$\hat{\pi}^{rw,rw}_t = \hat{p}^{rw,rw}_t - \hat{p}^{rw,rw}_{t-1} \quad (55)$$

$$\hat{p}^{rw,rw}_t = \widehat{rer}^{i,rw}_t + \hat{p}^{rw,i}_t \quad (56)$$

$$\widehat{mc}^{rw}_t = \sigma_L^{rw}\left(\hat{y}^{rw}_t - \frac{\varepsilon^{rw}_{a,t}}{1-\alpha_k^{rw}}\right) + \frac{\sigma_C^{rw}}{1-h^{rw}}\left(\hat{y}^{rw}_t - h^{rw}\hat{y}^{rw}_t\right) + \hat{\varepsilon}^{rw}_{L,t} + u^{rw}_{\mu_w,t} \quad (57)$$

$$\hat{r}^{rw}_t = \rho_r^{rw}\hat{r}^{rw}_t + (1-\rho_r^{rw})\left(\rho_{r\pi}^{rw}\hat{\pi}^{rw}_t + \rho_{ry}^{rw}\hat{y}^{rw}_t\right) + u^{rw}_{r,t} \quad (58)$$

Additionally, equations to describe investment from the rest of the world into country i and their accumulation:

$$\hat{x}_t^{rw,i} = \rho_{x^{rw}}^i \hat{x}_{t-1}^{rw,i} + u_{x^{rw},t}^i \quad (59)$$

$$\hat{k}_t^{rw,i} = (1 - \delta)\hat{k}_{t-1}^{rw,i} + \delta\hat{\varepsilon}_{X,t}^i + \delta\hat{x}_t^{rw,i} \quad (60)$$

where $\rho_{x^{rw}}^i$ is autoregressive coefficient; $u_{x^{rw},t}^i$ is i.i.d. white noise.

The designations of variables, parameters and shocks of the rest of the world are similar to the designations of the Customs Union member-countries. Shocks of the rest of the world $\hat{\varepsilon}_{C,t}^{rw}, \hat{\varepsilon}_{a,t}^{rw}, \hat{\varepsilon}_{L,t}^{rw}, u_{\mu_p,t}^{rw}, u_{\mu_w,t}^{rw}, u_{r,t}^{rw}$ are given similarly to shocks of the countries of the Customs Union.

III. Parameter Estimation of the Model

The solution of the linear Model (2) was obtained on the basis of the Blanchard-Kahn algorithm [14] and this solution is represented as the first-order vector autoregression:

$$\hat{X}_t = Q^\theta \hat{X}_{t-1} + F^\theta H_t^{\Sigma_H} \quad (61)$$

where $t = 1,2,\ldots$; Q^Φ, F^Φ are matrices of appropriate dimensions.

To estimate the parameters as the results of measurements of the observed variables there were taken the log-deviation (in percentage) from their trend values of the following macroeconomic indicators of the Customs Union: final consumption expenditures of households; final consumption expenditures of government; gross accumulation; exports of goods and services; GDP; consumer price index; average nominal wages; people employed in the economy; refinancing rate of the Central Bank; government budget revenues; government budget expenditures; tax revenues; domestic government debt; foreign government debt; the official exchange rate, the volume of oil production; and world data: OECD GDP; inflation in the OECD; short-term interest rate of the U.S. Federal Reserve. Statistical data is adjusted from seasonal components using the algorithm X-12-ARIMA [15] and detrended by the HP filter [16]. The interval of the parameters estimation is from the 1 quarter of 2000 till the 4 quarter of 2012.

Estimation of the parameters passed in two stages, using the Bayesian approach [17]. The essence of the Bayesian approach is to use a likelihood function obtained on the model (61) using the Kalman filter and the prior distribution of the parameters for finding the joint posterior density of parameter estimates distribution. Then, sampling for the posterior density, consisting of a large number of parameter sets is generated using the algorithm of Metropolis–Hastings. Corresponding average values of the samples were taken as required parameter estimates.

At the application of the Bayesian approach, standard for DSGE models a priori distribution density types of parameters Φ, Σ_H were used: i.i.d. white noise standard deviations are set by inverse gamma distributions, the limited parameters between 0 and 1 are set by beta distributions, the rest of parameters are set by normal distributions. At the first stage a priori probability characteristics of the parameters have been set according to [5] and taking into account the specific features of the economies, and parameter estimation of the economies of each country has been carried out separately (i.e. with fixed values of parameters of the other countries). At the second stage features of the posteriori estimates of the parameters received at the first stage were used to define the a priori distribution of the parameters and to obtain the final parameter estimates.

The solution of the linear model and the estimation of the parameters described in this section were obtained using the software Dynare MATLAB Toolbox [18].

The quality of parameter estimates was verified by retroforecast. The mean square deviation of the obtained forecasted values of the economic indicators from the relevant statistics is about 3%.

IV. Macroeconomic Analysis of the Shock Effects

The following figures show obtained research results on the effects of internal and external shocks on GDP and inflation of the national economy of Kazakhstan for the last 13-year time interval and the 10-year forecast time horizon.

Fig. 1 and Fig. 2 show decompositions of GDP and inflation deviations from the trend for the period from 1 quarter of 2000 till 4 quarter of 2012 in Kazakhstan under different shock effects.

Here lines show the values of macroeconomic statistics for GDP and inflation in terms of deviations from their trend values, and the columns show by what shocks these values formed.

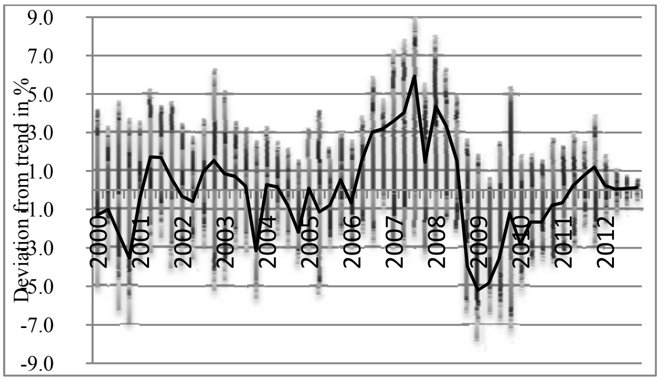

Fig. 1. Decomposition of GDP dynamics of Kazakhstan under internal and external shock effects.

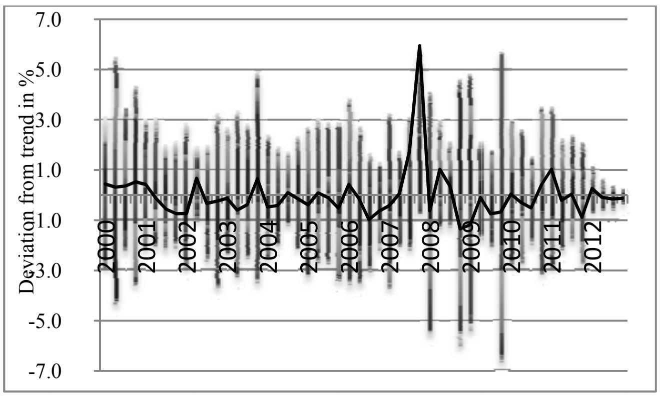

Fig. 2. Decomposition of inflation dynamics of Kazakhstan under internal and external shock effects.

Fig. 3 and Fig. 4 show decomposition of standard deviation of forecasted GDP and inflation by internal and external shock effects.

The shocks most strongly influencing consumer price inflation deviation from the forecast in the short term are: shock of oil production (up to 35% of standard deviation), productivity shock (up to 19.5% of standard deviation), shock of oil prices (up to 17% of standard deviation), interest rates shock (up to 11% of standard deviation), labor supply shock (up to 8.9% of the standard deviation).

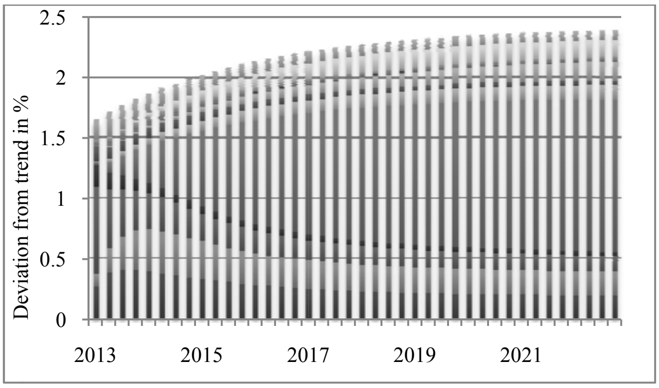

Fig. 3. Decomposition of standard deviation of forecasted GDP of Kazakhstan under internal and external shock effects.

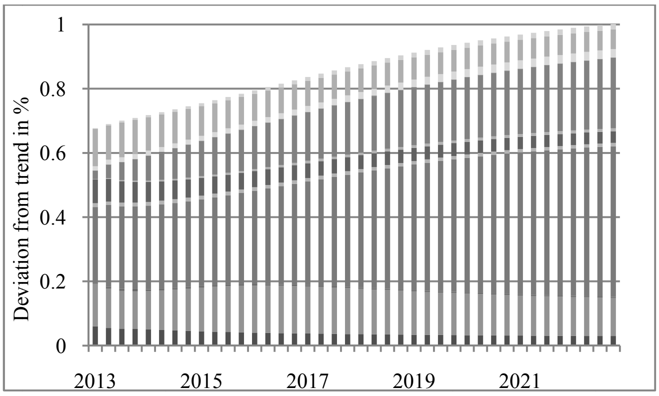

Fig. 4. Decomposition of standard deviation of forecasted inflation of Kazakhstan under internal and external shock effects.

The shocks most strongly influencing consumer price inflation deviation from the forecast in the midterm and the long term are: shock of oil production (up to 37% of standard deviation), preference shock (up to 22% of standard deviation), productivity shock (up to 12% of standard deviation), shock of oil prices (up to 6% of standard deviation), interest rate shock (up to 4% of standard deviation).

The shocks most strongly influencing GDP deviation from the forecast in the short term are: productivity shock (up to 16% of standard deviation), shock of oil prices (up to 43% of standard deviation), shock of oil production (up to 10% of standard deviation), interest rate shock (up to 6% of standard deviation), preference shock of the rest of the world (up to 8% of standard deviation).

The shocks most strongly influencing GDP deviation from the forecast in the midterm and the long term are: shock of oil production (up to 52% of standard deviation), productivity shock (up to 8% of standard deviation), preference shock of the rest of the world (up to 5% of standard deviation), labor supply shock (up to 7% of standard deviation), shock of foreign inflation (up to 3% of standard deviation), shock of oil prices (up to 5% of standard deviation).

The large deviation of GDP from trend values in 3 quarter of 2007 was mainly because of shock of oil production (3,7% points), preference shock of the rest of the world (-2,6% points), preferences shock (0,5% points), investment shock (0,5% points) and productivity shock (0,4% points).

The large deviation of consumer price inflation from steady state value in 4 quarter of 2007 was because of many shocks of the Model.

V. Parametric Control of Macroeconomic Indicators Volatility for the Economy of Kazakhstan on the Base of Model

The problem of parametric control of macroeconomic indicators volatility for the economy of Kazakhstan on the base of DSGE model can be formulated as follows:

$$\min_{u^{kz}_{r,t+1},\dots,u^{kz}_{r,t+40},u^{kz}_{ge,t+1},\dots,u^{kz}_{ge,t+40}} \mathrm{E}_t \sum_{i=kz}^{40} \beta^i \left({\hat{\pi}^{kz}_{C,t+i}}^2 + \lambda_Y {\hat{y}^{kz}_{t+i}}^2 + \lambda_R {\hat{r}^{kz}_{t+i}}^2 + \lambda_G {\hat{g}^{kz}_{t+i}}^2 \right) \quad (62)$$

under the following constraints $(i = 1,2,3,\dots)$

$$\hat{X}_{t+i} = Q^{\Phi} \hat{X}_{t+i-1} + F^{\Phi} H^{\Sigma_H}_{t+i} \quad (63)$$

$$\left|\mathrm{E}_t \hat{\pi}^{kz}_{C,t+i}\right| \le 0.5 \quad (64)$$

$$\left|\mathrm{E}_t \hat{r}^{kz}_{t+i}\right| \le 0.5 \quad (65)$$

$$\left|\mathrm{E}_t \hat{g}^{kz}_{t+i}\right| \le 5 \quad (66)$$

Here $u^{kz}_{r,t+1}, \dots, u^{kz}_{r,t+40}$ are additive terms (24), considered for the case of parametric control as estimated deterministic solutions of the formulated problem (62) – (66); $u^{kz}_{ge,t+1}, \dots, u^{kz}_{ge,t+40}$ are additive terms (25), considered for the case of parametric control, estimated as deterministic solutions of the formulated problem (62) – (66); $\hat{\pi}^{kz}_{C,t}, \hat{y}^{kz}_t, r^{kz}_t, \hat{g}^{kz}_t$ are deviations in percentage from their trend values for inflation, GDP, interest rate and government spendings, respectively; $\lambda_Y = 0.20$, $\lambda_R = 0.01$, $\lambda_G = 0.01$ are weight coefficients of objective function of the government economic policy; $\beta = 0.99$ is the discount factor.

Conditions (65), (66), as well as inclusion $\hat{r}^{kz}_{t+i}$, $\hat{g}^{kz}_{t+i}$ into criterion of formulated optimization problem define domain of search for optimal values of instruments $u^{kz}_{r,t+1}, \dots, u^{kz}_{r,t+40}, u^{kz}_{ge,t+1}, \dots, u^{kz}_{ge,t+40}$.

Formulated problem (62) – (66) was solved by numerical method.

Estimation of measures of government economic policy of Kazakhstan on the base of the parametric control theory was made for the period of previous 13 years (for 2000 – 2012) and the horizon of future 10 years (for 2013 – 2023).

In the first variant, the estimates of government policy in the national economy of Kazakhstan on the basis of the parametric control theory were carried out on the basis of results of model parameters estimates and the corresponding estimates of all considered shocks.

The actual and calculated values of GDP and inflation in mentioned interval are presented in Fig. 5. and Fig. 6.

Computation results show that, for this variant, parametric control of suppression of shock effects provides decrease of sampling standard deviation of GDP by 34% and inflation by 27% in comparison with actual values for mentioned period.

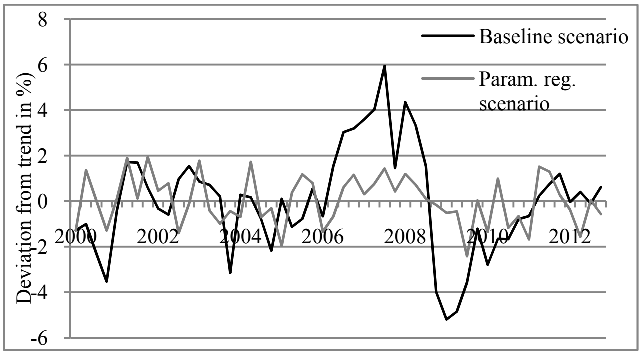

Fig. 5. Counterfactual analysis in interval 2000-2011, GDP (deviation from trend in %).

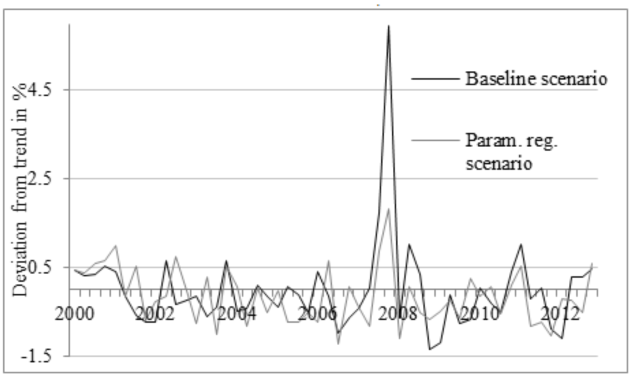

Fig. 6. Counterfactual analysis in interval 2000-2011, inflation (in % per quarter).

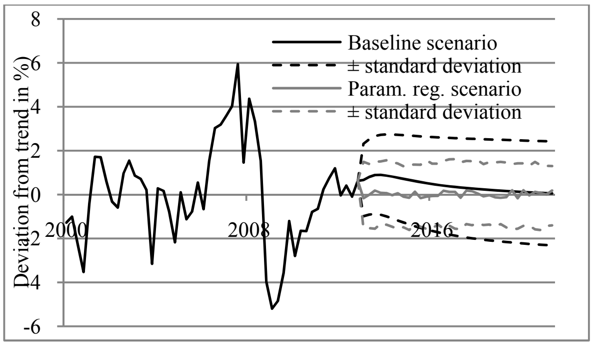

Fig. 7. GDP forecast with parametric control and basic scenario (deviation from trend in %).

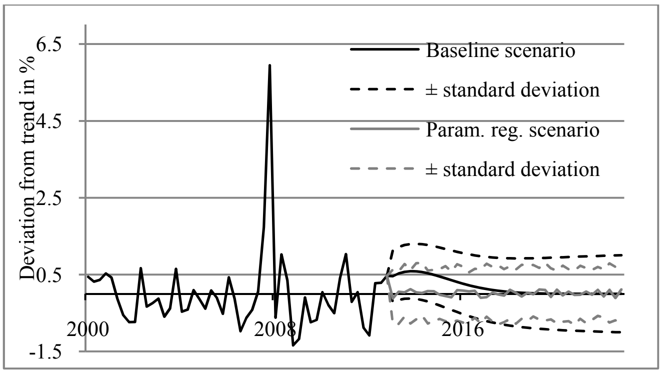

Fig. 8. Inflation forecast with parametric control and basic scenario (in % per quarter).

In the second variant, the problem of parametric control for minimizing GDP and inflation volatility of national economy of Kazakhstan was solved for the period of 2013-2023 as follows.

On the basis of the Model with parameter estimates obtained using statistical data for 2000-2012, there were estimated 500 trajectories by endogenous variables, including by generation of exogenous shocks for given autoregressive parts and by generation of their noise components by their characteristics, obtained from the Model estimation by statistical data for 2000-2012.

For each of 500 trajectories, the problem of parametric control was solved in the framework of the problem (62)-(66). Sample means of criteria and instruments are taken as the solution result of the parametric control problem.

Fig. 7 and Fig. 8 present sample means and sample standard deviations of GDP and inflation given using numerical solution of the problem by the Monte-Carlo method, as well as mathematical expectations and standard deviations of GDP and inflation, built on the base of the model (61) without using the parametric control approach. Computing results show that, for this variant, parametric control of macroeconomic indicators volatility provides decrease of forecast standard deviations of GDP by 35% and decrease of forecast standard deviations of inflation by 21% in average for mentioned forecasting horizon.

VI. Conclusion

DSGE model of the Customs Union and the rest of the world is constructed and estimated. Verification of the estimated DSGE model was made using retroforecast. Verification results demonstrated sufficient accuracy of the description of the functioning of national economies using this model.

Based on macroeconomic analysis, there were defined shocks affecting largely on GDP and inflation. The efficiency of solving the problem of minimizing the GDP and inflation volatility of the national economy of Kazakhstan by parametric control approach was shown.

These results can be used in the development and implementation of effective economic policy.

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