An Aggregate Production Function Explaining Negative Technological Shocks

Xianming Meng

***Abstract*—By introducing the growth rate of new-product innovations into the Solow growth model, this study displays how negative technological shocks could occur frequently and thus the production function can explain the economic growth and the business cycles at the same time.**

***Index Terms*—New products, economic growth, business cycle, technological shock, market saturation.**

I. Introduction

The endogenous growth models based on Solow growth model [1]-[4] may give some insight into the economic growth in the long run but are incapable of explaining the business cycles. On the other hand, the real business cycle theory [5], [6] may mimic the business cycles pretty well, but its explanation heavily relies on the frequent questionable negative technological shocks. This study provides an explanation for cyclical negative technological shocks and thus explains the economic growth and the business cycle at the same time.

II. The Revised Aggregate Production Function

The contribution of technology to economy can be classified into two types: one is the increase in the output for existing products and the other is the innovation of new products. The progress of technology should always positively affect the capacity of producing existing products, so the contribution of technology on output for existing products should not decrease over time (the bottom line is that zero technological progress will make zero contribution to the growth of production capacity). The innovation of new products creates the new demand for economy, so its contribution should be also positive. However, According to the product life cycle theory, a product in the market experiences four phases: introduction of a new product, growth, maturity and decline. If the speed of new-product innovations is not high enough, the market demand for old products will be saturated and thus the economic growth will be stagnant or even be negative due to the overproduction in the previous period. So, the speed of new-product innovation can affect economy positively or negatively. To embody the constraint of the speed of new-product innovations, we introduce into the neoclassical aggregate production function the growth rate of the number of new products as the exponent of technology level, shown as follows:

$$Y = A^{\frac{\Delta N}{N}} F(L,K)$$

where

Y – the total output
A – the technology level
N – the number of new products
ΔN – the change of the number of new products
L – the labour input
K – the capital input

This function shows that the total output level is determined by the level of technology, labour input and capital input, and the speed of innovation of new products. Due to the variation of the growth rate of new-product innovations, the effect of technology may be enlarged or reduced. Especially, since the change in the number of new products may be positive or negative, the effect of technology may magnify or lessen the effects of labour and capital. Moreover, the cyclical pattern of new product innovation may generate cyclical effect of technology change, which in turn results in the cyclical economic fluctuations – the business cycles.

III. The Growth Model

Based on the revised aggregate production function, we can derive the economic growth model.

Using the log form of the production function and fully differentiating it, we have:

$$\ln Y_t = \frac{\Delta N}{N} \ln A + \ln\left(F(L,K)\right)$$

$$dY_t \;/ Y = \frac{\Delta N}{N} dA \;/ A + (\ln A)\, d\frac{\Delta N}{N} + \frac{1}{F(L,K)} \frac{\partial F(L,K)}{\partial L} dL + \frac{1}{F(L,K)} \frac{\partial F(L,K)}{\partial K} dK$$

$$= \frac{\Delta N}{N} dA \;/ A + \frac{\Delta N(\ln A)}{N} d\left(\frac{\Delta N}{N}\right) / \frac{\Delta N}{N} + \frac{L}{F(L,K)} \frac{\partial F(L,K)}{\partial L} dL / L + \frac{K}{F(L,K)} \frac{\partial F(L,K)}{\partial K} dK / K$$

let

$$S_l = \frac{L}{F(L,K)} \frac{\partial F(L,K)}{\partial L}, S_k = \frac{K}{F(L,K)} \frac{\partial F(L,K)}{\partial K}$$

We have the following growth model:

$$dY_t / Y = \frac{\Delta N}{N} dA / A + \frac{\Delta N(\ln A)}{N} d\left(\frac{\Delta N}{N}\right) / \frac{\Delta N}{N} + S_l dL / L + S_k dK / K$$

If we measure the differentiations of variables as the year-on-year changes, the above equation indicates the relationship among the percentage annual growth rates of output, new products, technology change, and labour and

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capital inputs. Given the data on the growth rates of output, new products, and labour and capital inputs, we can estimate the contribution of labour and capital to the economy and, more importantly, the growth rate of technology (dA/A) and the base technology level (lnA).

IV. The Empirical Findings

There is a difficulty in applying the growth model to empirical estimation: the number of new products is hard to measure. However, in a modern economy (production capacity is not an issue), the sales growth rate is a good indicator of market potential or, put it in another way, market saturation. Since the influence of growth rate of product innovation in the aggregate production function is fulfilled through the restriction of market saturation, the sales growth rate is an ideal candidate to replace the growth rate of product innovations. In doing so, we obtain an empirical model for estimation:

$$OUTPUT = C1 + C2 * SAL + C3 * DSAL + C4 * CAP$$
$$+ C5 * LAB + \mu$$

where

OUTPUT – the percentage annual growth rate of GDP

SAL – the percentage annual growth rate of final sales (GDP minus inventory)

DSAL – the percentage annual growth rate of PSAL

CAP – the percentage annual growth rate of capital input

LAB – the percentage annual growth rate of labour input

The data used for estimation are annual data during 1966 to 2008 from the Australian Bureau of Statistics (ABS). The time series of growth rate of labour input and capital input are from the Productivity Table in Australian National Accounts (ABS 5204.0). The annual growth rates of GDP and final sales (GDP minus inventory) are calculated based on the data on GDP and inventory in Australian National Accounts.

The ADF and Perron unit root tests both suggest I(1) for OUTPUT, SAL and CAP and I(0) for other variables. Using the Johansen procedure [7], [8] to test the cointegration among OUTPUT, SAL and CAP, we find that both trace and max-eigenvalue tests suggest one cointegration. However, the small sample size (only 43) in this study may bias the tests – the asymptotic property of the Johansen test is not applicable. Using the adjusted critical value calculated according to the suggestion of Reimers [9] and Cheung and Lai [10], we find that the testing results remain the same. Since the cointegration among I(1) variables is confirmed, it is valid for us to estimate the model using the dynamic ordinary least square (DOLS) developed by Saikkonen [11] and generalised by Stock and Watson [12]:

$$OUTPUT_t = C1 + C2 * SAL_t + C3 * DSAL_t + C4 * CAP_t$$
$$+ C5 * LAB_t + \sum_{m=-M}^{M} (A_m \Delta SAL_{t-m} + B_m \Delta CAP_{t-m}) + \mu_t$$

To minimize SIC, 2 leads and 2 lags are used in the estimation. The estimation results are as follows (we omit the coefficients on leads and lags because the purpose of the use of leads and lags is to increase the estimation efficiency):

$$OUTPUT = -0.01 + 0.97 * SAL + 0.001 * DSAL + 0.32 * CAP + 0.18 * LAB + \mu$$

Wald Stat.	1.756	439.7	0.042	2.543	3.561
s. t.	0.008	0.046	0.007	0.199	0.099
p-value	0.185	0.000	0.837	0.111	0.059

R-squared=0.985, adjusted R-squared=0.973, D.W.=2.069

It is not surprising that the Durbin Watson (D.W.) statistic implies the existence of autocorrelation as DOLS allows for a Moving Average (MA) process in the residuals. Since the residuals are auto correlated, the high (adjusted) R-squared value is not reliable. However, Stock and Watson [12] demonstrates that the DOLS estimators have large-sample chi-squared distributions and thus the Wald test is applicable. Therefore, the above standard errors and the p-values from Wald tests are valid. The model passes all other diagnostic tests (e.g. the white heteroskedesticity test, the J.B. normality test, the recursive test, the CUSUM test, the CUSUM of square, etc.)

The DOLS estimators reveal the following interesting findings:

First, the growth rates of sales, capital input and labour input all have significant positive effects on the growth of GDP. The large Wald statistic for the coefficient of sales growth rate demonstrates its importance. The capital and labour inputs are significant at around 10% and 6% level respectively. The point estimates show that the capital input contributes about twice as much as the labour input does.

Second, the estimate of the coefficient for SAL implies that the technology growth rate in the concerned period is very high. According to the growth model, the coefficient of SAL indicates the technology growth rate. An average annual growth rate of around 97% (92%-101%) for more than 40 years has brought dramatic change to Australia.

Finally, the insignificance of DSAL is consistent with the theoretic growth model. Referring to the growth model, the coefficient of DSAL indicates the log value of technology level multiplied by the growth rate of the number of new products (or the sales growth rate in the empirical model). The log value is small; the growth rate of new products can be positive or negative, and should be very small due to the long innovation cycles. As a result, the average value of the sum of the products should be close to zero as shown by the estimation results.

Appendix

TABLE I: Results of Unit Root Tests*

Variable	Level of test	t-statistics (ADF test)	Conclusion of ADF test
OUTPUT	Level	-0.873	unit root
	First difference	-5.363	No unit root
CAPITAL	Level	-0.105	unit root
	First difference	-4.302	No unit root
LABOUR	Level	-0.917	unit root
	First difference	-4.177	No unit root
SALES	Level	-0.998	unit root
	First difference	-5.880	No unit root
DSALES	Level	-7.708	No unit root
	First difference	-6.16	No unit root

* The number of lags is chosen to minimize AIC.

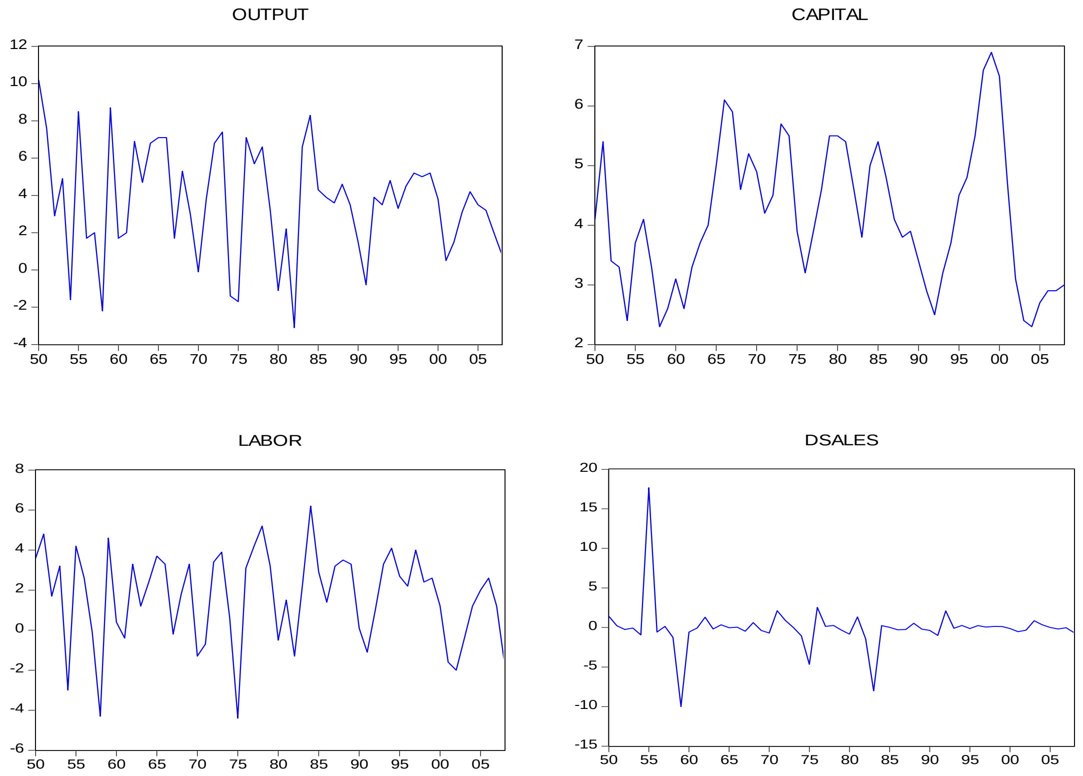

Fig. 1. Graphs of time series

TABLE II: RESULTS OF COINTEGRATION TESTS*

No. of CE(s)	Trace Statistic	0.05 Critical Value	Adjusted Critical Value	*No. of CE(s)*	Max-Eigen Statistic	0.05 Critical Value	Adjusted Critical Value
None *[a]	113.5713	69.81889	93.62078	**None *[a]**	48.0698	33.87687	45.4258
At most 1 *[a]	65.50153	47.85613	64.17072	**At most 1***	33.58319	27.58434	36.98809
At most 2 *	31.91834	29.79707	39.95516	**At most 2**	17.25813	21.13162	28.33558
At most 3	14.66021	15.49471	20.777	**At most 3**	11.30935	14.2646	19.12753
At most 4	3.350859	3.841466	5.151057	**At most 4**	3.350859	3.841466	5.151057

* denotes rejection of the hypothesis at the 0.05 level according to the **standard** critical value
[a] denotes rejection of the hypothesis at the 0.05 level according to the **adjusted** critical value
3 lags are chosen to minimize AIC.

TABLE III: RESULTS OF DOS ESTIMATION

Dependent Variable: OUTPUT
Method: Least Squares
Sample (adjusted): 1954 2005
Included observations: 52 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.189588	0.607592	-1.957872	0.0599
CAPITAL	0.100172	0.135143	0.741232	0.4645
LABOR	0.253555	0.116442	2.177511	0.0377
SALES	1.189288	0.107786	11.03379	0.0000
DSALES	0.003797	0.027604	0.137541	0.8916
D(CAPITAL(1))	0.473369	0.208577	2.269519	0.0309
D(LABOR(1))	-0.021761	0.107056	-0.203269	0.8403
D(SALES(1))	0.132699	0.110342	1.202620	0.2389
D(CAPITAL(-1))	-0.242764	0.183062	-1.326134	0.1951
D(LABOR(-1))	-0.267629	0.070839	-3.777974	0.0007
D(SALES(-1))	0.194115	0.075437	2.573194	0.0155

D(CAPITAL(2))	-0.204730	0.236850	-0.864387	0.3945
D(LABOR(2))	-0.051379	0.105937	-0.484996	0.6313
D(SALES(2))	0.130196	0.113720	1.144880	0.2616
D(CAPITAL(-2))	-0.166386	0.201694	-0.824944	0.4161
D(LABOR(-2))	-0.112401	0.082992	-1.354351	0.1861
D(SALES(-2))	0.048235	0.092348	0.522320	0.6054
D(CAPITAL(3))	0.125779	0.230227	0.546328	0.5890
D(LABOR(3))	0.116243	0.080737	1.439771	0.1606
D(SALES(3))	-0.186403	0.085615	-2.177220	0.0377
D(CAPITAL(-3))	-0.259732	0.161947	-1.603807	0.1196
D(LABOR(-3))	-0.000465	0.056135	-0.008291	0.9934
D(SALES(-3))	-0.079353	0.072552	-1.093736	0.2831

R-squared	0.984577	Mean dependent var	3.582692
Adjusted R-squared	0.972877	S. D. dependent var	2.949358
S.E. of regression	0.485731	Akaike info criterion	1.694346
Sum squared resid	6.842117	Schwarz criterion	2.557396
Log likelihood	-21.05300	Hannan-Quinn criter.	2.025219
F-statistic	84.15110	Durbin-Watson stat	2.068755
Prob(F-statistic)	0.000000		

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