Risk Neutral Density Estimation of Option Price from the Perspective of Empirical Analysis

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***Abstract*—Options markets are evolutionary, and the risk-neutral density (RND) provides some insight into the market's perception of the future direction of the underlying index and risk. At the same time, the risk-neutral density is also an important tool for analyzing the dynamics of the financial market, the attitudes, and reactions of traders, the phenomena that have been experienced to shock the financial market, and some of the potential shocks. This study not only takes the Chinese options market as the research object and uses the smoothing implied volatility smile curve method to select the 50 most liquid ETF options, to estimate the risk-neutral density, but also finds the relationship between RND and macroeconomic announcements. Since the risk-neutral density is provided by the option price of a given asset at different strike prices at the same time, it can reflect the market's evaluation of the solvency probability of a series of option strike prices. The study find that the time of macroeconomic announcements and the good news make influence to change RND. The announcement and good news on variance and kurtosis have an opposite effect in both the volatile period and post-crash period.**

***Index Terms*—SSE 50ETF option, risk neutral density, implied volatility, volatility smile curve.**

I. Introduction

Risk neutral density, which is provided by the price of options sold at the same time on the expiration date for specific assets at different strike prices and can indicate the market's assessment of the probability of repayment at a range of strike prices. A good estimation of risk-neutral density is an uncertain problem and remains a mathematical and computational challenge. Parametric and nonparametric methods for estimating risk-neutral density from option prices have been developed and applied in the literature and industry.

To extract a well-behaved risk-neutral density from a set of option prices, two problems need to be addressed. Firstly, the theory requires that the strike price of the option be continuous. Secondly, another problem is that by solving the first problem with interpolation and smoothing methods, we can only extract the middle part of the density, and since the range of the strike price is very small, it cannot be further extended to the two tails, deviate from market expectations, theoretically the density will not change. By studying RND, we can get market belief. And RND outperforms implied volatility, because implied volatility is a measure of the second moment of the underlying price, whereas RND is all the moments. In addition, the evolution of risk-neutral density can shed light on how to market beliefs change over time. It might be thought of as examining how to market beliefs change scheduled or unscheduled macroeconomic announcements.

The nonparametric approach uses more general functions to gain greater flexibility and uses certain criteria to accommodate option prices. A well-known method of this class is the smoothing implied volatility smile method (SML). The smoothing implied volatility smile method by Figlewski (2009) [1] was adopted to extract the risk-neutral density. SML takes advantage of the fact that the second derivative of the option price function is proportional to the RND. Different approaches have been proposed to accommodate implied volatility, such as Shimko (1993) [2]. A simple quadratic polynomial is proposed to fit the volatility and strike price within Shimko's existing data points. MalzShimko's method is modified to use the option of $\delta = \partial C$ to fit the implied volatility, rather than the strike price of ∂S. He argues that implied volatility corresponding to delta options is more accurate than implied volatility corresponding to strike options. Campa, Chang, and L. S. Rompolis (2006) [3] propose to use natural splines, rather than low-order polynomials, to fit implied volatility against strike prices. Natural splines can control the smoothness of the fitting function and increase the flexibility of the model. They propose to use smoothed cubic splines to fit implied volatility relative to option increment.

From the risk-neutral density, main approaches to examine the impact of macroeconomic news announcements on financial markets including the GARCH model, like Beber and Brandt (2006) [4]; Hess, Huang and Niessen (2008) [5]. Beber and Brandt (2006) compare the option-implied moments before and after the announcements and find that the announcements reduce the uncertainty on all news, which is consistent with the study Jiang, Konstantinidi and Skiadopoulos (2012) [6].

Adopting the double lognormal method to extract the RND, Gemmill and Saflekos (2000) [7] find that RND does not help to reveal investors' sentiment during British elections. With regard to the Chinese financial market, although, Baum, Kurov and Wolfe (2015) [8] try to study how the Chinese scheduled macroeconomic announcements influenced on the global financial and commodity futures markets. Tang *et al*. (2013) [9] examine the impact of monetary policies, including the changes of interest rate and the required reserve ratio, on the Chinese stock markets, rare can be found through Chinese option market.

Because this method attempts to smooth the implied volatility space with Black-Scholes formula rather than option price, it is more linear and smoother than option price,

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so it can obtain better interpolation results. Then, the continuum fitting the implied volatility is converted back to the continuum fitting the option price. Finally, the implied risk neutral density can be obtained by Equations (1) and (2):

$$f(S_T) = e^{rT}\frac{\partial^2 C}{\partial X^2} \quad (1)$$

$$f(S_T) = e^{rT}\frac{\partial^2 P}{\partial X^2} \quad (2)$$

II. Estimating Risk Neutral Density

A. Calculate Implied Volatility

The option data of 50 ETFs is January 23, 2020 option data. See Table I in Appendix for the specific data. All data variables, including option prices, risk-free interest rates, dividend yields and the price levels of 50 ETFs, are derived from uqer.datayes.com.

The Black-Scholes formula for European call options is as follows:

$$C = S_t N(d_1) - Xe^{-rT}N(d_2) \quad (3)$$

$$P = Xe^{-rT}N(-d_2) - S_t N(-d_1) \quad (4)$$

Parameters are: d_1 d_2

$$d_1 = \frac{\ln\left(\frac{S_t}{X}\right) + \left(r + \frac{\delta^2}{2}\right)^{\tau}}{\delta\sqrt{\tau}}.$$

$$d_2 = \frac{\ln\left(\frac{S_t}{X}\right) + \left(r - \frac{\delta^2}{2}\right)^{\tau}}{\delta\sqrt{\tau}}.$$

where C is the European call option price, K is the strike price, T is the time of the transaction, R is the risk-free interest rate, δ is the dividend yield, τ is the current stock price, N is the standard normal cumulative distribution, and σ is the volatility of the stock (the standard deviation of the logarithmic return of the stock). Every parameter in the Black-Scholes formula is known in the market, except volatility σ, which is also known as implied volatility.

Finally, processing the relevant data on January 23 to get the price curve, the final price curve is shown in Fig. 1, and the implied volatility curve is shown in Fig. 2:

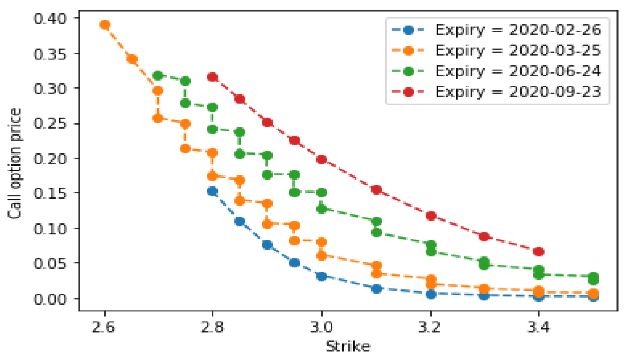

Fig. 1. Price curve.

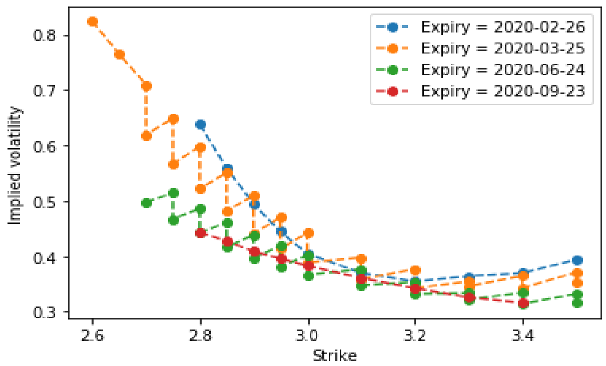

Fig. 2. Implied volatility curves.

B. Derivation of Risk-Neutral Density

In the risk-neutral valuation method Cox and Ross, the price of the European call option of the stock is expressed as follows:

$$w = \frac{X_{high} - X}{X_{high} - X_{low}}. \quad (5)$$

where F (S) is RND, K is the strike price, S is the share price at maturity, T is the time to maturity, and R is the risk-free interest rate.

The derivative of the execution price K is as follows:

$$\begin{aligned}\frac{\partial C(K)}{\partial K} &= (S-K)f(S)|_{S=K} + e^{-rt}\int_K^{\infty}\frac{\partial (S-K)f(S)}{\partial K}dS \\ &= 0 + e^{-rt}\int_K^{\infty} -f(S)dS \\ &= -e^{-rt}\int_K^{\infty} f(S)dS\end{aligned} \quad (6)$$

Then differentiate the execution price K:

$$\begin{aligned}\frac{\partial^2 C(K)}{\partial K^2} &= e^{-rt}f(S)|_{S=K} \\ &= e^{-rt}f(K)\end{aligned} \quad (7)$$

Hence RND such as Breeden and Litzenberger Shown below:

$$f(K) = e^{rt}\frac{\partial^2 C(K)}{\partial K^2} \quad (8)$$

C. The Role of the Smoothing Implied Volatility Smile Curve Method

The method of smoothing the implied volatility smile curve was originally proposed by Shimko, which smooths the implied volatility space calculated by the Black-Scholes formula. This method tries to smooth the implied volatility smile curve it can get interpolation results.

Due to the nature of option price, this method can solve these two problems. On the one hand, it can solve the smoothness of risk-neutral density. On the other hand, the theory states that the strike price of an option is a continuum from zero to infinity. However, this is not the case in the market, so interpolation and smoothing are needed to solve these problems.

D. An Introduction to the Method of Smoothing the Implied Volatility Smile Curve

Among the non-parametric methods for estimating RND, the smoothed implied volatility smile method (SML) is noted for its simplicity of implementation. To further illustrate SML, we will briefly review the concept of the volatility smile. The main idea of SML is summarized as follows:

Firstly, the Black-Scholes formula is used to convert the available call option prices into implied volatility. Secondly, using implied volatility that meets specific criteria. Thirdly, using polynomial smoothing to interpolate the implied volatility curve. Finally, interpolate the mixed implied volatility through a quaternary polynomial smoothing. The correlative implied volatility with the deadline of February 23, 2020, is known, so the smile curve of correlative volatility can be obtained, as shown in Fig. 3.

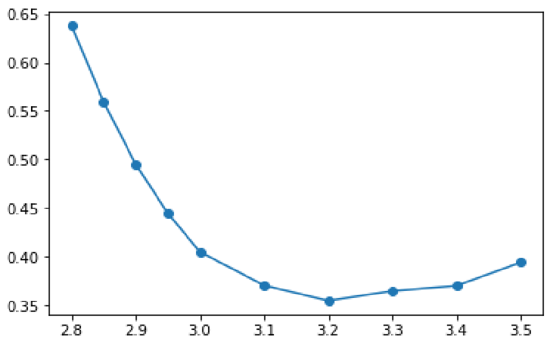

Fig. 3. Volatility smile curve.

The following is the implied volatility curve after I used fourth-order interpolation fitting and smoothing, as shown in Fig. 4.

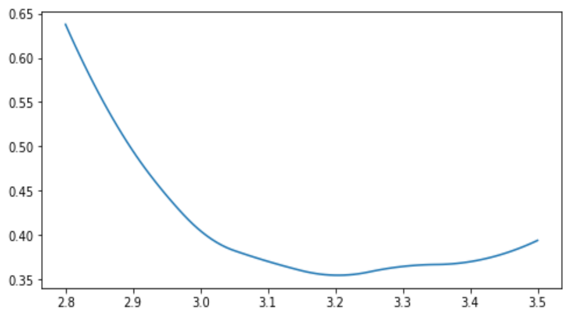

Fig. 4. Volatility curve after fourth-order interpolation fitting.

The Black-Scholes formula is used again to convert the fitted implied volatility back to the option price function. Calculate the second derivative of the option price function to estimate RND. The smoothed volatility smile curve is used to plot the estimated risk-neutral density, as shown in Fig. 5

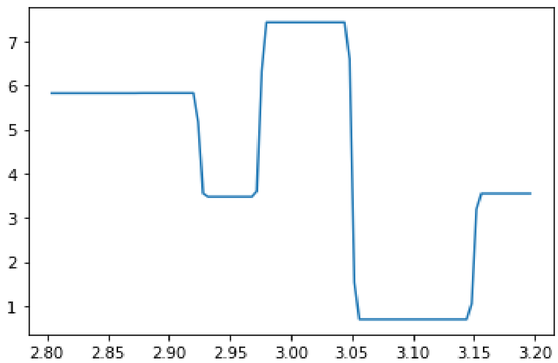

Fig. 5. Estimated risk-neutral density.

III. The Reaction of Option Prices to Macroeconomic Announcements: Evidence from China's 50 ETF Options

A. 50ETF

Fig. 6. 50ETF option data.

The 50ETF option data is from September 11, 2020, to March 19, 2021. Fig. 6 gives the data provided by the Shanghai Stock Exchange, the performance of China's 50ETF options can be seen in the stable operation of China's financial market.

B. Release of China's Macroeconomic News

The release of the macroeconomic news announcements is scheduled in advance, such as Consumer Price Index, Gross Domestic Product, Foreign Exchange Reserves, etc. Calling the release of macroeconomic news announcements are not pre-scheduled (Unscheduled) announcements, including the change of the base rate and the change of the Reserve Requirement Ratio. Furthermore, a positive asset return comes from a piece of good news. In other words, if this statistic is higher than the last one, it is called good news. Here, for some announcements that are released at weekends or on holidays, regarding the news as good news if the return is positive in the following trading day. Moreover, considering a zero return as a sign of good news.

TABLE I: SAMPLE MACROECONOMIC NEWS ANNOUNCEMENTS IN CHINA

Date	Classifications	Event Name	Unit	Actual	Good or bad news
Jan-21	National Account	GDP	%	-8.1	bad
Mar-21	Surveys& Cyclical	Manufacture PMI – HSBC PMI Flash	%	10	good
Mar-21	Surveys& Cyclical	PMI Manufacturer-NBS Manufacturing PMI	–	52.2	good
Dec-20	External Sector	Trade-Exports YY	%	3.6	good
Dec-20	External Sector	Trade-Imports YY	%	-1	bad
Dec-20	External Sector	Trade Balance	USDB	53.5	good
Oct-20	Prices	Inflation - PPI YY	%	-0.8	bad
Mar-21	Prices	Inflation-CPI YY	%	0.6	good

Mar-21	Government Sector	Money and lending-New Yuan Loans	CNYB	8654	good
Mar-21	Surveys & Cyclical	Money and lending – Outstanding Loan Growth	%	0.8	good
Mar-21	National Account	Activity indicators-Urban investment	%	25.6	good
Mar-21	Industry Sector	Industrial Output	%	14.1	good
Mar-21	Consumer Sector	Activity indicators – Retail Sales YY	%	34.2	good
Mar-21	Surveys& Cyclical	PMI-HSBC PMI Flash	–	51.9	good
Mar-21	Surveys& Cyclical	PMI-NBS Manufacturing PMI	–	53.9	good
Feb-21	Surveys &Cyclical	PMI-HSBC PMI Flash	_	50.6	bad
Feb-21	Surveys &Cyclical	PMI-NBS Manufacturing PMI	–	51.9	bad

For description, several macroeconomic announcements (see samples in Table I) have been picked. By the count, there were 12 good news and 5 bad news, for a total of 17 macroeconomic announcements. The good news is concentrated in March 2021 and the bad news is concentrated in January 2021 and February 2021.

C. Estimating the Risk-Neutral Density

Estimating the risk-neutral density by smoothing the implied volatility smile curve. The movement of the means corresponds to the series of the China 50 ETF prices, which is reasonable because the means are always approximate to the expected value of the underlying price under the no-arbitrage condition. This is consistent with the findings by Figlewski (2009) and Fabozzi, Leccadito, and Tunaru (2014) [10]. Theoretically, the change in the risk-neutral variance measures the resolution of uncertainty.

D. The Time-Varying Behavior of Risk-Neutral Moments

At first, figuring out the first moment of RND. The corresponding moments for the densities can be found in the following Fig. 7:

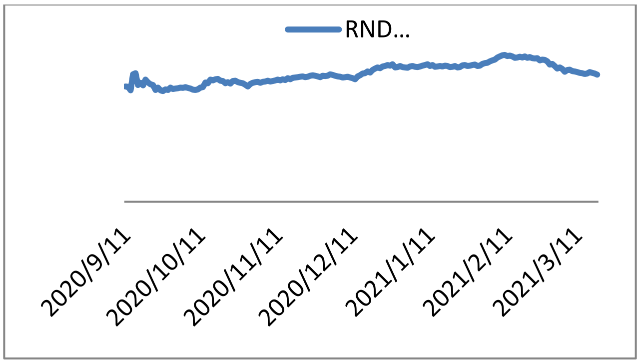

Fig. 7. First moment of RND.

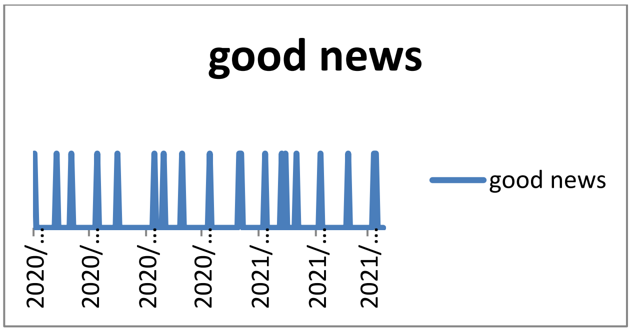

Fig. 8. Good news.

Then, figure out the good news and announcement. The results can be found in Fig. 8 and Fig. 9. And the dummies description is shown in Table II:

TABLE II: DUMMIES DESCRIPTION

	0	1
Announcements dummy	No announcement	Have announcement
Good dummy	Bad news	Good news

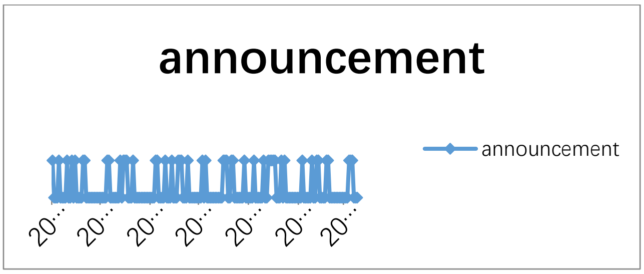

Fig. 9. Announcement.

Then, regarding macroeconomic announcement as a time series of a dummy variable, and the value is 1 when there is an announcement and 0 when there is no announcement. Regression assessing the contemporaneous effect:

$$\Delta moment_t = \alpha \Delta announce + \beta \Delta good\ news + \varepsilon_t$$

As we can see from Fig. 6, a huge crash was taken place on 10 Feb 2021. We find that the date on 10 Feb is a significant breakpoint for all four moments over the sample period ($p < 0.05$). Therefore, I prefer to design the regression consist of the whole period, pre-crash period, and post-crash period. What we are interested in is the change of the moments. Then we can find the results in Table III.

TABLE III: CHANGES IN MEAN

Regressions for moments of RND Dependent variable: Changes in Mean

	Whole period	Pre-crash	Post-crash
Announce	0.0452	0.0791	0.0328
	(0.0332)	(0.104)	(0.0315)
L.Announce	-0.0523*	-0.153	-0.0349*
	(0.0413)	(0.412)	(0.0325)
Good news	0.0517*	0.0436	0.0361*
	(0.0332)	(0.0662)	(0.0217)
L. good news	0.0652**	0.136*	0.0381*
	(0.0246)	(0.0623)	(0.0235)
N	190	83	107
R-sq	0.124	0.167	0.192
Adj.R-sq	0.092	0.03	0.332
rmse	0.0856	0.139	0.0639

Standard errors in parentheses
*=p < 0.1, **= p < 0.05, ***= p < 0.01

The regression of the changes in the first moment shows how does mean changes concerning the news over the sample period and two sub-sample periods. For the whole sample period, the density shifts in the case of the announcement, if it is good news. The announcement yesterday and the good news of today would decrease the changes in the mean of the density for the whole period and the post-crash period. Both good news today and yesterday would increase the mean of the density. In other words, these would shift the density to the right, but the news today for the regression is not significant in pre-crash. The results can be found in Table IV.

TABLE IV: CHANGES IN THE SECOND MOMENT

Regressions for moments of RND Dependent variable: Changes in the second moment			
	Whole period	Pre-crash	Post-crash
Announce	-0.0833 (0.0712)	-0.611** (0.194)	0.0633*** (0.0315)
L.Announce	0.0123 (0.0602)	0.267 (0.372)	-0.0132 (0.0322)
Good news	-0.0797 (0.0592)	-0.00571 (0.1662)	-0.0263* (0.0227)
L. good news	-0.106* (0.0546)	-0.262* (0.132)	0.082 (0.0221)
N	190	83	107
R-sq	0.06	0.264	0.173
Adj.R-sq	0.009	0.143	0.083
rmse	0.199	0.285	0.06

Standard errors in parentheses
*=p < 0.1, **= p < 0.05, ***= p < 0.01

According to the regression of the changes in the second moment, the good news yesterday decreased the variance today. This is not surprising because the release of the good news will decrease the overall uncertainty of the market for the whole period and the pre-crash period. But we did not find a significant effect in the post-crash period. Comparing with the effect of the announcement and the good news have the opposite effect in the volatile period and post-crash period. The results can be found in Table V.

TABLE V: CHANGES IN THE THIRD MOMENT

Regressions for moments of RND Dependent variable: Changes in the third moment			
	Whole period	Pre-crash	Post-crash
Announce	0.0152	-2.42	0.433**
	(0.532)	(1.942)	(0.15)
L.Announce	-0.602	-1.267	-0.326*
	(0.602)	(2.342)	(0.172)
Good news	-0.197	-0.310	-0.183
	(0.467)	(1.262)	(0.172)
L. good news	-0.161	-0.342	0.0082
	(0.436)	(1.132)	(0.221)
N	190	83	107
R-sq	0.033	0.064	0.173
Adj.R-sq	-0.029	-0.043	0.183
rmse	1.189	2.256	0.461

Standard errors in parentheses
*=p < 0.1, **= p < 0.05, ***= p < 0.01

About the regression of the changes in the third moment, we did not find any effect for the whole period and the pre-crash period. In the post-crash period, the announcement, their lags have a significant effect on the skewness. The results can be found in Table VI.

TABLE VI: CHANGES IN THE FOURTH MOMENT

Regressions for moments of RND Dependent variable: Changes in the fourth moment			
	Whole period	Pre-crash	Post-crash
Announce	-0.132	4.77**	-1.433**
	(0.714)	(2.942)	(0.615)
L.Announce	0.657	1.667	0.576
	(0.712)	(3.035)	(0.575)
Good news	0.167	-1.108	0.0183
	(0.661)	(1.452)	(0.672)
L. good news	0.756	1.142	0.188
	(0.646)	(1.362)	(0.621)
N	190	83	107
R-sq	0.042	0.145	0.143
Adj.R-sq	-0.019	-0.013	0.033
rmse	1.989	2.698	1.461

Standard errors in parentheses
*=p < 0.1, **= p < 0.05, ***= p < 0.01

The fourth moment measures both the highest value of the density. The results decrease the kurtosis of the density. We also find the different effects of the announcement and the surprise in the pre-crash and post-crash period.

IV. CONCLUSION

In this paper, by using the smoothing implied volatility smile method to extract the risk-neutral density of more than 50 ETF option prices. The contribution here is threefold: The first contribution is that many papers summarize existing methods to find risk-neutral density, but few review the application of risk-neutral density. Second, identified the best method to extract the risk-neutral density. In other words, few studies have compared the performance of Figlewski's smoothing implied volatility smile method with other alternative methods. Thirdly, this paper also studies the relationship between the moment implied by the risk-neutral density and the current value of the underlying object.

This paper also investigates the impact of macroeconomic news announcements on China’s 50 Exchange-Traded Fund (50 ETF) options, by distilling the information with the risk-neutral densities extracted from the market.

In the Chinese market, for the whole sample period, the density changes in the case of the announcement and the good news. The good news yesterday falls the variance today. And the good news makes the kurtosis lower. In the pre-crash period, the macroeconomic announcements do have some effect on the densities. The good news would increase the mean of the density. The good news does increase the uncertainty of the market. In contrast, the announcement and the good news result in the fall of the variance. In the post-crash period, we also find that the announcement yesterday and surprise today show an increase in the downside risk. The announcement and good news on variance and kurtosis have an opposite effect in both the volatile period and post-crash period. However, there is no effect of the macroeconomic news announcement on the skewness in the whole period and the pre-crash period.

Conflict of Interest

The authors declare no conflict of interest".

Author Contributions

Rui Ren conducted the research, wrote the paper; Rongchen Wang analyzed the data.

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