A Method for Risk Assessment from Natural Disasters Using an Actuarial Model

Plamena Zlateva and Dimiter Velev

***Abstract*—The paper proposes a method for risk assessment from natural hazards using an actuarial model. In particular the method is based on the collective risk model for evaluation of the consequences /aggregated loss due to occurrence of the natural hazard with certain intensity level and probability in the considered time interval. The risk assessment results can support the state and local government to take more informed decisions regarding the efficient allocation of the available funding for the improvement of the community and infrastructure from natural hazards. The proposed risk assessment method with actuarial model is envisaged to be implemented as a part of an information system for risk management of natural disasters. This system can be successfully used in e-government.**

***Index Terms*—Risk assessment, actuarial model, collective risk model, natural hazard.**

I. Introduction

Over the last ten year time frame, the natural disasters have continued to affect strongly negatively to the well-being and safety of persons, communities, economy, environment, infrastructure and etc. [1], [2]. The annual losses resulting from floods, hurricanes, droughts**,** earthquakes, tornadoes, etc. cost billions of dollars.

Natural disasters are difficult to avoid. For these reasons United Nations (UN), European Union (EU) and any governments develop strategies to disaster risk reduction in regard to the increased natural disasters [3]-[5]. European Commission in the end of 2010 proposed official working paper "Risk Assessment and Mapping Guidelines for Disaster Management" [6].

Hence there is a need to propose new and modified methods for risk assessment from natural disasters. The availability of an adequate assessment of the aggregated loss / consequences due to occurrence of natural hazards would help taking more informed decisions for effective risk management.

The purpose of the paper is to propose a method for risk assessment from natural disasters using an actuarial model. In particular the method is based on the collective risk model for evaluation of the consequences /aggregated loss due to occurrence of the natural hazard with certain intensity level and probability in the considered time interval.

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II. Essence of the Risk Assessment Process

Risk management standards are a useful tool in representing and logically organizing risk management process in a way that makes decision-making open to inputs from different stakeholders [7], [8]. In Fig. 1 is shown contribution of risk assessment to the risk management process according EN 31010:2010 "Risk management - Risk assessment techniques" (IEC/ISO 31010:2009) [8].

The risk assessment is the overall process of risk identification, risk analysis and risk evaluation.

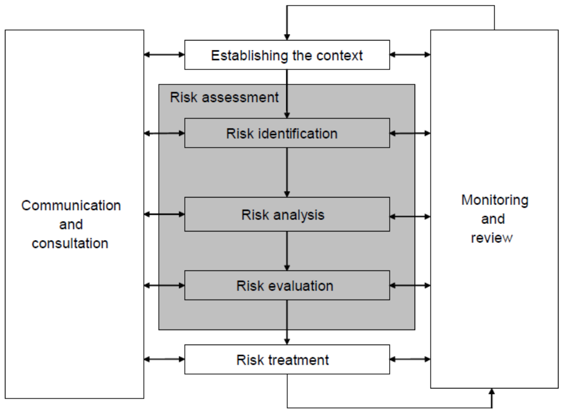

Fig. 1. Contribution of risk assessment to the risk management process.

It is necessary to point that the risk assessment may require a multidisciplinary approach since risks from natural hazards ordinary cover a wide range of causes and consequences.

Usually for the purpose of risk assessment from natural hazards the following term definitions are used [6], [8], [9]:

- *Hazard* is a dangerous phenomenon, substance, human activity or condition that may cause loss of life, injury or other health impacts, property damage, loss of livelihoods and services, social and economic disruption, or environmental damage.
- *Natural hazard*: Natural process or phenomenon that may cause loss of life, injury or other health impacts, property damage, loss of livelihoods and services, social and economic disruption, or environmental damage. Natural hazard events can be characterized by their magnitude or intensity, speed of onset, duration, and area of extent.

- *Exposure*: People, property, systems, or other elements present in hazard zones that are thereby subject to potential losses.
- *Vulnerability*: The characteristics and circumstances of a community, system or asset that make it susceptible to the damaging effects of a hazard. In probabilistic/ quantitative risk assessments the term vulnerability expresses the part or percentage of *Exposure* that is likely to be lost due to a certain hazard.
- *Risk* is a combination of the consequences of an event (*Hazard*) and the associated likelihood/probability of its occurrence.
- *Risk assessment* is the overall process of risk identification, risk analysis, and risk evaluation.
- *Risk identification* is the process of finding, recognizing and describing risks.
- *Risk analysis* is the process to comprehend the nature of risk and to determine the level of risk.
- *Risk evaluation* is the process of comparing the results of risk analysis with risk criteria to determine whether the risk and/or its magnitude is acceptable or tolerable.
- *Risk criteria* are the terms of reference against which the significance of a risk is evaluated.
- *Consequences* are the negative effects of a disaster expressed in terms of human impacts, economic and environmental impacts, and political/social impacts.
- *Human impacts* are defined as the quantitative measurement of the following factors: number of deaths, number of severely injured or ill people, and number of permanently displaced people.
- *Economic and environmental impacts* are the sum of the costs of cure or healthcare, cost of immediate or longer-term emergency measures, costs of restoration of buildings, public transport systems and infrastructure, property, cultural heritage, etc., costs of environmental restoration and other environmental costs (or environmental damage), costs of disruption of economic activity, value of insurance pay-outs, indirect costs on the economy, indirect social costs, and other direct and indirect costs, as relevant.
- *Political/social impacts* are usually rated on a semi-quantitative scale and may include categories such as public outrage and anxiety, encroachment of the territory, infringement of the international position, violation of the democratic system, and social psychological impact, impact on public order and safety, political implications, psychological implications, and damage to cultural assets, and other factors considered important which cannot be measured in single units, such as certain environmental damage.
- *Threat* is a potentially damaging physical event, phenomenon or activity of an intentional/ malicious character.
- *Single-risk assessments* determine the singular risk (i.e. likelihood and consequences) of one particular hazard (e.g. flood) or one particular type of hazard (e.g. flooding) occurring in a particular geographic area during a given period of time.
- *Multi-risk assessments* determine the total risk from several hazards either occurring at the same time or shortly following each other, because they are dependent from one another or because they are caused by the same triggering event or hazard; or merely threatening the same elements at risk (vulnerable/ exposed elements) without chronological coincidence.
- *Hazard assessments* determine the probability of occurrence of a certain hazard of certain intensity.

According IEC/ISO 31010:2009 [8] in situations where the likelihood of occurrence of a hazard of certain intensity can be quantified investigators refer to the term probability of occurrence. When the extent of the impacts is independent of the probability of occurrence of the hazard, which is often the case for purely natural hazards, such as earthquakes or storms, risk can be expressed algebraically as [6]:

*Risk = probability of occurrence * hazard impact*

or

$$R = f\left(P \times C\right),$$

where R is risk; P - probability of occurrence of the natural hazard*;* C – consequences (natural hazard impact).

Risk matrix is very helpful in the risk assessment process. In particular the risk matrix or so-called consequence/ probability matrix is a means of combining qualitative or semi-quantitative ratings of consequence and probability to produce a level of risk.

The format of the risk matrix depends on the context in which it is used. The scale used may have 5 or more points. The matrix may be set up to give extra weight to the impact or to the likelihood, or it may be symmetrical.

Usually in risk assessment process the risk matrix 5x5 is used (Fig. 2).

Here the *Probability levels* (Relative likelihood) are graded as "*Very low*", "*Low*", "Medium", "*High*" and "*Very high*". *Consequences* (Relative impact) are also graded as "*Very low*", "*Low*", "*Medium*", "*High*" and "*Very high*".

The *Risk levels*, *R* are defined as "*Low*", "*Medium*", "*High*" and "*Very high*".

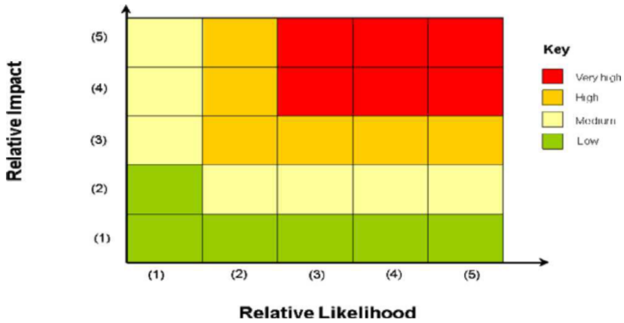

Fig. 2. Risk matrix 5×5: Consequences (relative impact) / probability (relative likelihood).

The following three main types of impacts in regard to the risk assessment are defined in [6]: human, economic/environmental, political/social. Within each category of impact (human, economic/environmental, political/social) the relative importance can be graded using a single set of criteria to score the relative likelihood and the relative impact applicable to the different hazards or risk scenarios.

In particular, the human impact can be estimated in terms of number of affected people and the economic/ environmental impact can be measured in currency (for example in Euro). The political/ social impact can be measured in a qualitative/ semi-quantitative scale comprising a number of classes, for example five classes as (1) limited/ insignificant, (2) minor/ substantial, (3) moderate/ serious, (4) significant/ very serious, (5) catastrophic/ disastrous [6].

The overall risk assessment requires producing distinct risk matrices for human impact, economic / environmental impact and political/social impact. However it is necessary to point that these categories are measured with distinct scales. For this reason it is very difficult to compare.

III. Description of the Proposed Method for Risk Assessment Based on Actuarial Model

The essence of the proposed method for risk assessment is consists in the evaluation of consequences /aggregated loss levels by an actuarial model. In particular the collective risk model is used.

Furthermore it is necessary to note that the adequate risk assessment requires producing distinct risk matrices for each intensity levels of the natural hazard. Usually it is considered four levels of the natural hazard intensity: (1) Low hazard intensity, (2) Medium hazard intensity, (3) High hazard intensity, (4) Very high hazard intensity (Table I).

TABLE I: Hazard Intensity Levels

Relative intensity	Hazard intensity levels, *H*
(4)	Very high hazard intensity
(3)	High hazard intensity
(2)	Medium hazard intensity
(1)	Low hazard intensity

A. Review of the Collective Risk Model

The collective risk model computes the aggregate loss as an independent sum of all losses incurred over a certain period [10]:

$$S = X_1 + X_2 + \ldots + X_N$$

where S is the aggregate loss; N - the number (frequency) of losses; X_i - the severity of the *i*-th loss, for $i = 1,\ldots,N$.

Commonly it is assumed that the loss severities X_i, $i = 1,\ldots,N$ are independently and identically distributed (iid) as the loss severity random variable X . The loss-frequency N is itself a nonnegative integer-valued random variable distributed independently of X_i, $i = 1,\ldots,N$.

The aggregate loss S is assumed to follow a nonnegative compound distribution. The loss-frequency random variable N represents the primary distribution and the loss-severity random variable X is secondary distribution of the compound distribution. Furthermore, *N* and *X* are assumed to be independent.

There are some advantages in modeling loss-frequency and loss-severity separately, and then combining them to determine the aggregate-loss distribution. Usually, primary and secondary distributions are determined by non-negative discrete random variables. In practice for the computation of the aggregate-loss distribution are used both recursive and approximate methods.

B. Main Properties of Compound Distributions

A few main properties of compound distributions are described below [10]-[12].

The moment generating function (mgf) of the random variable X as a function of *t* is denoted by $M_X(t)$. If the expectation exists then it is defined as follow

$$M_X(t) = \mathrm{E}\left(e^{tX}\right)$$

If the $M_X(t)$ exists for *t* in an open interval around $t = 0$, then the moments of X exist and can be obtained by successively differentiating the $M_X(t)$ with respect to *t* and evaluating the result at $t = 0$.

The *r*-th derivative of the $M_X(t)$ is described by the following dependence:

$$M_X^r(t) = \frac{d^r M_X(t)}{dt^r} = \frac{d^r}{dt^r}\mathrm{E}\left(e^{tX}\right) =$$

$$= \mathrm{E}\left[\frac{d^r}{dt^r}\left(e^{tX}\right)\right] = \mathrm{E}\left(X^r e^{tX}\right).$$

Therefore, at the point $t = 0$ for the *r*-th derivative of the $M_X(t)$ is obtained

$$M_X^r(0) = \mathrm{E}\left(X^r\right) = \mu_r^{'},$$

where by definition *r*-th initial moment of the random variable X in the general case is given by the formula

$$\mathrm{E}\left(X^r\right) = \sum_{i=1}^{\infty} x_i^r . f_{X_i}(x_i) = \mu_r^{'}$$

The probability generating function (pgf) of the nonnegative random variable X is denoted by $\mathrm{P}_X(t)$. If the expectation exists then it is defined as follow

$$\mathrm{P}_X(t) = \mathrm{E}\left(t^X\right).$$

It is known that the mgf $M_X(t)$ and pgf $\mathrm{P}_X(t)$ are related through the following equations

$$M_X(t) = \mathrm{P}_X\left(e^t\right).$$

The moment generating function (mgf) of the aggregate loss S as a function of *t* is denoted by $M_S(t)$.

If the primary distribution N has mgf $M_N(t)$ and the secondary distribution X has mgf $M_X(t)$, then the mgf of the compound distribution S is deduce as follow

$$M_S(t) = E\left(e^{tS}\right) = E\left(e^{t.(X_1+X_2+...+X_N)}\right) =$$

$$= E\left(E\left(e^{t.X_1+t.X_2+...+t.X_N}\,|N\right)\right) = E\left(\left(E\left(e^{t.X}\right)\right)^N\right) =$$

$$= E\left(\left(M_X(t)\right)^N\right) = E\left(\left(e^{\ln M_X(t)}\right)^N\right) = M_N\left(\ln M_X(t)\right)$$

If N has pgf $\mathrm{P}_N(t)$ and X is nonnegative integer valued with pgf $\mathrm{P}_X(t)$, then the pgf of S is

$$P_S(t) = E\left(t^S\right) = E\left(E\left(t^{X_1+X_2+...+X_N}\right)|N\right) =$$

$$= E\left(\left(E\left(t^X\right)\right)^N\right) = E\left(\left(P_X(t)\right)^N\right) = P_N\left(P_X(t)\right).$$

Furthermore, the aggregate loss S is nonnegative and discrete, because the loss-severities take nonnegative discrete values. The mean of the aggregate loss S is given by

$$ES = E\left(E\left(S|N\right)\right) = E\left(E\left(N.X\right)|N\right) = E\left(N.E\left(X\right)\right) =$$

$$= E\left(N.\mu_X\right) = \mu_X E\left(N\right) = \mu_N \mu_X = EN.EX\ .$$

The variance of the random variables S is defined using the condition of independence between random variables X and N, and the following relations:

$$D\left(S\right) = E\left(S^2\right) - \left(ES\right)^2 \quad \text{and} \quad E\left(S^2\right) = E\left(E\left(S^2|N\right)\right);$$

$$E\left(X|N\right) = EX \quad \text{and} \quad D\left(X|N\right) = DX\ .$$

Thus the variance of the aggregate loss S is is deduce as follow

$$D(S) = E\left(D\left(S|N\right)\right) + E\left(\left(E\left(S|N\right)\right)^2\right) - \left(E\left(E\left(S|N\right)\right)\right)^2 =$$

$$= E\left(ND\left(X|N\right)\right) + D\left(NE\left(X|N\right)\right) =$$

$$= E\left(N.\sigma_X^2\right) + D\left(N.\mu_X\right) = \mu_N.\sigma_X^2 + \sigma_N^2.\mu_X^2\ .$$

The collective risk model is described only theoretically. Further research is needed particularly regarding its implementation for aggregate loss assessment (Aggregated loss levels / Consequences) of the monitored object due to natural hazards for a certain time interval.

C. Risk Assessment Using the Collective risk Model

In this study the total risk assessment of a monitored object from natural hazard with four intensity levels for a certain time interval is defined as

$$R = \sum_{k=1}^{4} R_k \tag{1}$$

where R_k is the risk assessment corresponding to k intensity level of natural hazard, k=1,...,4.

The risk assessment R_k is obtained by following product

$$R_k = P_k.C_k \qquad k=1,...,4 \tag{2}$$

where P_k is the occurrence probability of the natural hazard with k intensity level; C_k is the consequences (the aggregate loss) caused by action of the of the natural hazard with k intensity level.

Here, it is proposed the consequences (the aggregate loss) of the monitored object caused by action of the natural hazard with k intensity level for a certain time interval to calculate as the collective risk model:

$$C_k = X_{1k} + X_{1k} + \ldots + X_{Nk} \tag{3}$$

where N is the number of losses from the natural hazard with k intensity level in the considered time interval; X_{ik} is the severity of the i loss from the natural hazard with k intensity level in the considered time interval for $i = 1,\ldots,N$.

The calculated value of consequences / aggregated loss levels, C_k by (3) and the given value of the probability P_k for occurrence of the natural hazard with k intensity level in the considered time interval are substituted in (2) to calculated corresponding risk level R_k.

Than using (1) it is calculated the total risk assessment R of a monitored object from natural hazard with four intensity levels for a certain time interval.

Each of the resulting risk assessments R and R_k, k=1,...,4 can be presented as a distinct risk matrix (Fig. 2).

First from Table II it can be determined the particular level of the probability P_k by using predefined range of each of the five levels. The constants IP_i , i=1,…5 are previously given.

Second by analogy with the probability from Table III it can be determined the particular level of the aggregated loss levels / consequences C_k by using predefined range of each of the five levels. The constants IC_i, i=1,…5 are previously given.

TABLE II: RELATIVE LIKELIHOOD / PROBABILITY LEVELS

Relative likelihood	Probability levels, P_k	Probability value intervals
(1)	Very low probability	$IP_4 < P_k$
(2)	Low probability	$IP_3 < P_k \le IP_4$
(3)	Medium probability	$IP_2 < P_k \le IP_3$
(4)	High probability	$IP_1 < P_k \le IP_2$
(5)	Very high probability	$P_k \le IP_1$

TABLE III: THE AGGREGATED LOSS / CONSEQUENCES LEVELS

Relative impact	Aggregated loss / Consequences levels, C_k	Consequence value intervals
(1)	Very low probability	$IC_4 < C_k$
(2)	Low probability	$IC_3 < C_k \le IC_4$
(3)	Medium probability	$IC_2 < C_k \le IC_3$
(4)	High probability	$IC_1 < C_k \le IC_2$
(5)	Very high probability	$C_k \le IC_1$

Third from Table IV for each of the resulting risk assessments R and R_k, k=1,...,4, can be determined the particular risk levels by using predefined range of each of the four levels. The constants IR_i , i=1,…4 are previously given.

The proposed risk assessment with these determinations of the particular risk levels R and R_k,, k=1,...,4, leads to more effectiveness of the risk management about natural hazards.

TABLE IV: THE RISK LEVELS

Risk levels, R or R_k	Risk value intervals
Very high risk	$IR_3 < R_k$
High risk	$IR_2 < R_k \leq IR_3$
Medium risk	$IR_1 < R_k \leq IR_2$
Low risk	$R_k \leq IR_1$

IV. CONCLUSION

The risk assessment as an essential part of risk management from natural disaster is a significant problem of the present day. Experts permanently offer new or modified methods for assessment of risk from natural disasters.

This study proposed a method for risk assessment from natural hazards using an actuarial model. In particular the method is based on the collective risk model for evaluation of the consequences /aggregated loss due to occurrence of the natural hazard with certain intensity level and probability in the considered time interval.

The proposed risk assessment method with actuarial model is envisaged to be implemented as a part of an information system for risk management of natural disasters. This system can be successfully used in e-government.

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