A Mathematical Bioeconomic Model of a Fishery: Profit Maximization of Fishermen

S. Bakht, Y. EL Foutayeni, and N. Fatmi Idrissi

***Abstract*—The present work joins in a scientific context which returns within the framework of the modelling and of the mathematical and IT analysis of models in dynamics of populations. In particular, it deals with the application of the mathematics and with the computing in the management of fisheries. In this work, we define a bio-economic model in the case of two marine species whose natural growth is modeled by a logistic law. These two marine species are exploited by two fishermen. The objective of the work is to find the fishing effort that maximizes the profit of each fisherman by using Nash equilibrium and taking into account constraints related to the conservation of biodiversity.**

***Index Terms*—Bio-economic model, maximizing profits, generalized Nash equilibrium GNE, linear complementarity problem LCP.**

I. Introduction

There exist very many elaborate mathematical models according to various parameters, and make it possible to make projections on the evolution of the fisheries and stocks of the marine species [1]-[4].

The models can then be categorized in two parts, those purely biological which do not take into account the economic interests, and those bioeconomic which integrate the output and the benefit of the fishermen [5]-[11].

In this work, we propose a model of two fishermen acting in an area containing two marine fish species. The evolution of fish populations is described by a density-dependent model, taking into account the competition between fishers (see the model of Verhulst [12]). More specifically, the bio-economic model consists of three parts: A biological part that links catch to biomass stock, a part of exploitation that links catch to fishing effort at equilibrium and an economic part that links effort fishing for profit.

The objective of each fisherman is to maximize his income without consulting the other by respecting two constraints: the first is the sustainable management of resources; the second is the preservation of biodiversity. With all these considerations, our problem leads to Nash equilibrium problem, to solve this problem, we transform it into a linear complementarity problem.

This work is organized as follows. In Section 1, we present a Mathematical study of the bioeconomic model of one marine species caught by one or two fishermen. Seeking to find the fishing effort that maximizes the profit of each fisherman taking into account constraints related to the conservation of biodiversity. In Section II we define the mathematical model of two marine species exploited by two fishermen. In Section III, we compute the Nash equilibrium point. In Section IV, we give a numerical simulation of the mathematical model and the discussion of the results. Finally, we conclude with a conclusion.

II. Bioeconomic Model for One Marine Species

In this section we consider the simple case of one marine species.

The interest lies in the study of a bioeconomic problem of a one marine specie exploited by one fisherman, which is characterized and presented by the equation

$$\dot{X} = rX\left(1 - \frac{X}{K}\right) - qEX \tag{1}$$

where r is the intrinsic growth rate, K is the carrying capacity, E is the fishing effort to exploit the one marine species by the fisherman and q is the catchability coefficient of marine species.

Our goal is to calculate the effort E that maximize the fisherman's profit π

$$\pi \quad E \quad = pH - cE \tag{2}$$

where p is the price of the fish population and $pH = pqEX$ is the total revenue.

At equilibrium we will have $X = K \;\; 1 - \frac{q}{r} E \;\;$.

Then $$\pi \quad E \quad = -K\frac{pq^2}{r}E^2 + \;\; pqK - c \;\; E$$

According to π is a second-order function with $-K\frac{pq^2}{r} < 0$, then π has a unique maximum E^* given by

$$E^* = \frac{r}{2}\left(\frac{1}{q} - \frac{c}{Kpq^2}\right).$$

The interest in the second part of this section concerns the study of a bioeconomic problem of one species exploited by two fishermen following the equation

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right) - H_1 - H_2 \tag{3}$$

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S. Bakht is with ENSA of Khouribga University Hassan 1st in Settat, Morocco (e-mail: bakht498@gmail.com)

Y. El Foutayeni is with Hassan II University of Casablanca, Morocco.

N. Fatmi Idrissi is with Hassan I University of Settat, Morocco.

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where $H_i = qE_iX$ for $i = 1,2$.

Our objective is to find $E(E_1, E_2)$ which maximize the profit $\pi(\pi_1, \pi_2)$ of the two fishermen.

At equilibrium we will have $X = K\left(1 - \frac{1}{r}q(E_1 + E_2)\right)$.

Then the first fisherman must solve the problem

$$\max \pi_1(E) = -\frac{K}{r}pq^2E_1^2 + \frac{K}{r}\left(rpq - \frac{c_1}{K}r - pq^2E_2\right)E_1$$
$$\text{subject to}\begin{cases} q_1E_1 + q_2E_2 \le r \\ E_1 > 0 \\ E_2 \text{ is given} \end{cases} \qquad \text{(P1)}$$

and the second fisherman must solve the problem

$$\max \pi_2(E) = -\frac{K}{r}pq^2E_2^2 + \frac{K}{r}\left(rpq - \frac{c_2}{K}r - pq^2E_1\right)E_2$$
$$\text{subject to}\begin{cases} q_1E_1 + q_2E_2 \le r \\ E_2 > 0 \\ E_1 \text{ is given} \end{cases} \qquad \text{(P2)}$$

The objective is to calculate the fishing effort which maximize the profit of each fisherman during the biological equilibrium for the two problems (P1) and (P2).

We recall that the point $E^*(E_1^*, E_2^*)$ is called Nash equilibrium point if and only if E_1^* is a solution of problem (P1) for E_2^* given, and E_2^* is solution of problem (P2) for E_1^* given.

The essential conditions of Karush-Kuhn-Tucker applied to the problem (P1) and (P2) require that if E_1^* is a solution of the problem (P1) and if E_2^* is a solution of the problem (P2), then there exist constants $m_1, m_2, v \geqslant$ such that

$$\begin{pmatrix} m_1 \\ m_2 \\ v \end{pmatrix} = \begin{bmatrix} 2\frac{K}{r}pq & \frac{K}{r}pq & -\frac{K}{r} \\ \frac{K}{r}pq & 2\frac{K}{r}pq & -\frac{K}{r} \\ -q & -q & 0 \end{bmatrix}\begin{pmatrix} E_1^* \\ E_2^* \\ 0 \end{pmatrix} + \begin{pmatrix} -Kp + \frac{c_1}{q} \\ -Kp + \frac{c_2}{q} \\ r \end{pmatrix}$$

This problem is equivalent to the linear complementarity problem $LCP(M,b)$ where

$$M = \begin{bmatrix} 2\frac{K}{r}pq_1 & \frac{K}{r}pq_2 & -\frac{K}{r} \\ \frac{K}{r}pq_1 & 2\frac{K}{r}pq_2 & -\frac{K}{r} \\ -q_1 & -q_2 & 0 \end{bmatrix}$$

and

$$b = \begin{pmatrix} -Kp + \frac{c_1}{q} \\ -Kp + \frac{c_2}{q} \\ r \end{pmatrix}$$

The matrix M is a P-matrix, then LCP(M,b) have one solution given by

$$\begin{cases} E_1^* = \frac{r}{pK}\frac{c_1}{q^2} \\ E_2^* = \frac{r}{pK}\frac{c_2}{q^2} \end{cases}$$

III. Bioeconomic Model for Two Marine Species

There are many mathematical models developed according to different parameters and allow to make projections on the evolution of the fishery and the stocks of the marine species.

The models can then be categorized into two parts, purely biological ones that do not take economic interests into account, and bioeconomic ones that integrate the returns and returns of fishermen.

A. Biological Model

The biological factors that play a role in the dynamics of these populations are none other than the rates of growth of this population which includes birth and death as well as individual movements.

According to the Malthus population dynamics model (see G. F. Gause [13]):

$$\begin{cases} \frac{dX_1(t)}{dt} = r_1X_1\left(1 - \frac{X_1}{K_1}\right) - c_{12}X_1X_2 \\ \frac{dX_2(t)}{dt} = r_2X_2\left(1 - \frac{X_2}{K_2}\right) - c_{21}X_1X_2 \end{cases} \qquad (4)$$

where r_i is the stock growth rate for $i = 1,2$, K_i is the system load capacity for $i = 1,2$, X_i is the population density $i = 1,2$ and $c_{ij}\ (1 \le i \ne j \le 2)$ Coefficient of competition between species i and species j .

$$\begin{cases} X_1^* = \frac{r_2K_1(r_1 - c_{12}K_2)}{r_1r_2 - c_{12}c_{21}K_1K_2} \\ X_2^* = \frac{r_1K_2(r_2 - c_{21}K_1)}{r_1r_2 - c_{12}c_{21}K_1K_2} \end{cases}$$

This solution may give the coexistence of the two species of fish, in which case the biomasses of the two species of fish are positive with $r_1 - c_{12}K_2 > 0$ and $r_2 - c_{21}K_1 > 0$.

B. Equilibrium Analysis

The steady state solutions are the solutions of the equations

$$\begin{cases} r_1X_1\left(1 - \frac{X_1}{K_1}\right) - c_{12}X_1X_2 = 0 \\ r_2X_2\left(1 - \frac{X_2}{K_2}\right) - c_{21}X_1X_2 = 0 \end{cases} \qquad (5)$$

This system of equations has eight solutions $P_1(0,0),\ P_2(K_1,0),\ P_3(0,K_2)$ and $P_4(X_1^*, X_2^*)$ where

$$\begin{cases} X_1^* = \frac{r_2K_1(r_1 - c_{12}K_2)}{r_1r_2 - c_{12}c_{21}K_1K_2} \\ X_2^* = \frac{r_1K_2(r_2 - c_{21}K_1)}{r_1r_2 - c_{12}c_{21}K_1K_2} \end{cases}$$

The variational matrix of the system (4) is

$$J=\begin{bmatrix} r_1\left(1-\dfrac{2X_1}{K_1}\right)-c_{12}X_2 & -c_{12}X_1 \\ -c_{21}X_2 & r_2\left(1-\dfrac{2X_2}{K_2}\right)-c_{21}X_1 \end{bmatrix}$$

Lemma 1: The point $P_1(0,0)$ is unstable.

Proof: The variational matrix of the system (4) at the steady state $P_1(0,0)$ is

$$J_1=\begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}$$

The eigenvalues of J_1 are $\lambda_1=r_1$ and $\lambda_2=r_2$. Then, the point $P_1(0,0)$ is unstable.

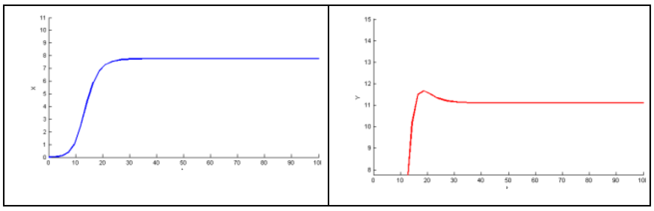

Fig. 1. Behaviors and phase portrait of system (4) at $P_1(0,0)$ .with the initial value $x(0)=0.01$, $y(0)=0.01$. The steady state P_1 of system (4) is unstable. Here $r_1=0.5$, $r_2=0.6$, $K_1=10$, $K_2=15$, $c_{12}=0.01$ and $c_{21}=0.02$.

Lemma 2: The point $P_2(K_1,0)$ is unstable.

Proof: The variational matrix of the system (4) at the steady state $P_2(K_1,0)$ is

$$J_2=\begin{bmatrix} -r_1 & -c_{12}K_1 \\ 0 & r_2-c_{21}K_1 \end{bmatrix}$$

The eigenvalues of J_2 are $\lambda_1=-r_1$ and $\lambda_2=r_2-c_{21}K_1$. Then, the point $P_2(K_1,0)$ is unstable.

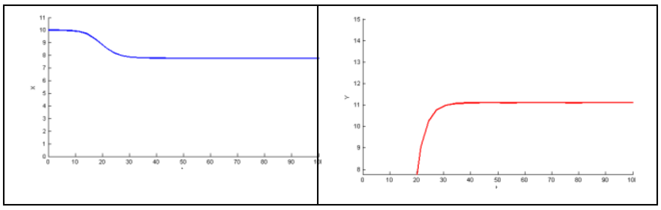

Fig. 2. Behaviors and phase portrait of system (4) at $P_2(10,0)$ .with the initial value $x(0)=9.9$, $y(0)=0.01$. The steady state P_2 of system (4) is stable. Here $r_1=0.5$, $r_2=0.6$, $K_1=10$, $K_2=15$, $c_{12}=0.01$ and $c_{21}=0.02$.

Lemma 3: The point $P_3(0,K_2)$ is unstable.

Proof: The variational matrix of the system (4) at the steady state $P_3(0,K_2)$ is

$$J_3=\begin{bmatrix} r_1-c_{12}K_2 & 0 \\ -c_{21}K_2 & -r_2 \end{bmatrix}$$

The eigenvalues of J_3 are $\lambda_1=r_1-c_{12}K_2$ and $\lambda_2=-r_2$.

Then, the point $P_3(0,K_2)$ is unstable.

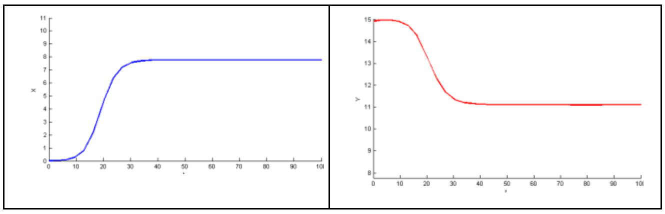

Fig. 3. Behaviors and phase portrait of system (4) at $P_3(0,15)$ .with the initial value $x(0)=0.01$, $y(0)=14.9$. The steady state P_3 of system (4) is stable. Here $r_1=0.5$, $r_2=0.6$, $K_1=10$, $K_2=15$, $c_{12}=0.01$ and $c_{21}=0.02$.

Lemma 4: The point $P_4(X_1^*,X_2^*)$ is stable.

Proof: The variational matrix of the system (4) at the steady state $P_4(X_1^*,X_2^*)$ is

$$J_4=\begin{bmatrix} -r_1\dfrac{X_1^*}{K_1} & -c_{12}X_1^* \\ -c_{21}X_2^* & -r_2\dfrac{X_2^*}{K_2} \end{bmatrix}$$

$trace(J_4)=-r_1\frac{X_1^*}{K_1}-r_2\frac{X_2^*}{K_2}<0$ and $det(J_4)=\left(\frac{r_1}{K_1}\frac{r_2}{K_2}-c_{12}c_{21}\right)X_1^*X_2^*>0$.

Then, the point $P_4(X_1^*,X_2^*)$ is stable.

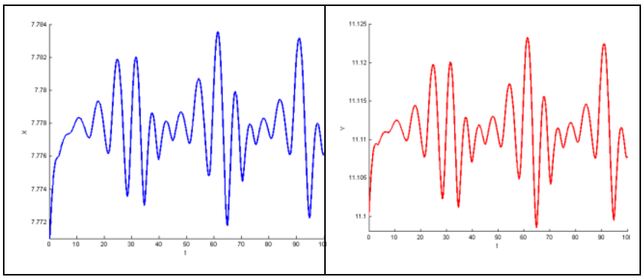

Fig. 4. Behaviors and phase portrait of system (4) at $P_4(7.776189158, 11.10779185)$ .with the initial value $x(0)=7.77$, $y(0)=11.10$. The steady state P_4 of system (4) is stable. Here $r_1=0.5$, $r_2=0.6$, $K_1=10$, $K_2=15$, $c_{12}=0.01$ and $c_{21}=0.02$.

C. Bioeconomic Model

The model for the evolution of fish population becomes

$$\begin{cases} X_1=r_1X_1\left(1-\frac{X_1}{K_1}\right)-c_{12}X_1X_2-q_1E_1X_1 \\ X_2=r_2X_2\left(1-\frac{X_2}{K_2}\right)-c_{21}X_1X_2-q_2E_2X_2 \end{cases} \tag{6}$$

where $(q_j)_{j=1,2,3}$ are the catchability coefficients of species j; and $(E_j)_{j=1,2,3}$ are the fishing effort to exploit a species j.

The catchability coefficient q is a key parameter in the validation process of fishing simulation model (see [14]). In this paper this parameter is assumed to be constant.

The fishing effort is defined as the product of a fishing activity and a fishing power. The fishing effort deployed by a fleet is the sum of these products over all fishing units in the fleet. The fishing activity is in units of time. The fishing power is the ability of a fishing unit to catch fish and is a complex function depending on vessel, gear and crew. However, since measures of fishing power may not be available, activity (such as hours or days fished) has often been used as a substitute for effort.

It is interesting to note that according to the literature, the effort depends on several variables, namely for example: Number of hours spent fishing; search time; number of hours since the last fishing; number of days spent fishing; number of operations; number of sorties flown; ship, technology, fishing gear, crew, etc. However, in this paper, the effort is treated as a unidimensional variable which includes a combination of all these factors. Now we give the expression of biomass as a function of fishing effort.

The biomasses at biological equilibrium are the solutions of the system

$$\begin{cases} r_1(1-\frac{X_1}{K_1}) = c_{12}X_2 + q_1E_1 \\ r_2(1-\frac{X_2}{K_2}) = c_{21}X_1 + q_2E_2 \end{cases} \tag{7}$$

The solutions of this system are given by

$$\begin{cases} X_1 = a_{11}E_1 + a_{12}E_2 + X_1^* \\ X_2 = a_{21}E_1 + a_{22}E_2 + X_2^* \end{cases} \tag{8}$$

where, $a_{11} = -K_1r_2q_1 / \Delta$, $a_{12} = K_1K_2q_2c_{12} / \Delta$, $a_{21} = K_1K_2c_{21}q_1 / \Delta$ and $a_{22} = -r_1K_2q_2 / \Delta$.

Or in matrix form $X = -AE + X^*$ where $A = (-a_{ij})_{1\leqslant i,j\leqslant 3}$ and $E = (E_1, E_2)^T$.

It is natural to assume that $r_jr_k > c_{ij}c_{ji}K_iK_j$ for all $j,k = 1,2$ which implies that $a_{ii} < 0$ for all $i = 1,2$.

The profit for each fisherman $\pi_i(E)$ is equal to total revenue $(TR)_i$ minus total cost $(TC)_i$, in other words, the profit for each fisherman is represented by the following function

$$\pi_i(E) = (TR)_i - (TC)_i$$

We use, as usual in the bioeconomic models, the fact that the total revenue (TR) depends linearly on the catch, that is, *Total revenue* = *Price* x *Catches*

As mentioned previously, we note that $H_{ij} = q_jE_{ij}X_j$ Catches of species j by the fisherman i, where E_{ij} is the effort of the fisherman i to exploit the species j. It is clear that $H_j = \sum_{i=1}^{n} H_{ij}$ is the total catches of species j by all fisherman.

On the other hand, we denote by $E_j = \sum_{i=1}^{n} E_{ij}$ the total fishing effort dedicated to species j by all fisherman and by $E^i = (E_{i1}, E_{i2})^T$ the vector fishing effort must provide by the fisherman i to catch the three species.
With these notations we have

$$(TR)_i = \sum_{j=1}^{2} p_jH_{ij} = < E^i, -pqAE^i > + < E^i, pqX^* - \sum_{j=1, j\neq i}^{2} pqAE^j > \tag{9}$$

where $(p_j)_{j=1,2,3}$ is the price per unit biomass of the species j. In this work, we take p_1 and p_2 to be constants.

We shall assume, in keeping with many standard fisheries models (e.g., the model of Clark [1] and Gordon [5]), that $(TC)_i = < c, E^i >$, where $(TC)_i$ is the total effort cost of the fisherman i, and $(H_j)_{j=1,2}$ = constant cost per unit of harvesting effort of species j.

As mentioned previously, the net economic revenue of each fisherman is represented by the following function $\pi_i(E) = (TR)_i - (TC)_i$.

It follows that

$$\pi_i(E) = < E^i, -pqAE^i > + < E^i, pqX^* - c - \sum_{j=1, j\neq i}^{2} pqAE^j > \tag{10}$$

As we have mentioned previously, the biological model is meaningful only insofar as the biomass of all the marine species are strictly positive (conservation of the biodiversity), then we have $X = -AE + X^* \geqslant 0$. In other words, for the fisherman i

$$AE^i \leqslant X^* - \sum_{j=1, j\neq i}^{n} AE^j. \tag{11}$$

IV. Nash Equilibrium

In this section, we restrict our self to the case when we have only two fishermen. For this case we can solve analytically the problem and give the solutions in explicit form.

Each fisherman trying to maximize his profit and achieve a fishing effort that is an optimal response to the effort of the other fishermen. We have a generalized Nash equilibrium where each fisherman's strategy is optimal taking into consideration the strategies of all other fishermen. A Nash Equilibrium exists when there is no unilateral profitable deviation from any of the fishermen involved. In other words, no fisherman would take a different action as long as every other fisherman remains the same. This problem can be translated into the following two mathematical problems:

The first fisherman must solve problem (P_1):

$$(P_1)\begin{cases} \max \pi_1(E) = < E^1, -pqAE^1 > + < E^1, pqX^* - c - pqAE^2 > \\ \text{subject to} \\ \qquad AE^1 \leqslant -AE^2 + X^* \\ \qquad E^1 \geqslant 0 \\ \qquad E^2 \text{ is given.} \end{cases}$$

and the second fisherman must solve problem (P_2):

$$(P_2)\begin{cases} \max \pi_2(E) = < E^2, -pqAE^2 > + < E^2, pqX^* - c - pqAE^1 > \\ \text{subject to} \\ \qquad AE^2 \leqslant -AE^1 + X^* \\ \qquad E^2 \geqslant 0 \\ \qquad E^1 \text{ is given.} \end{cases}$$

We recall that (E^1, E^2) s called Generalized Nash equilibrium point if and only if E^1 is a solution of problem $(P1)$ for E^2 given, and E^2 is a solution of problem $(P2)$ for E^1 given.

For solving the Nash equilibrium problem we use the essential conditions of Karush-Kuhn-Tucker. These conditions applied to the problem (P_1) and to the problem (P_1), require that if E^1 is a solution of the problem (P_1) and if E^2 is a solution of the problem (P_2), then there exist constants $u^1,\ u^2,\ v,\ \lambda^1,\ \lambda^2 \in \mathbb{R}^2_+$ such that

$$\begin{cases} u^1 = 2pqAE^1 + c - pqX^* + pqAE^2 + A^T\lambda^1 \\ u^2 = 2pqAE^2 + c - pqX^* + pqAE^1 + A^T\lambda^2 \\ v = -AE^1 - AE^2 + X^* \\ < u^i, E^i > = < \lambda^i, v > = 0 \qquad \text{for all } i = 1,2 \\ E^i, u^i, \lambda^i, v \geqslant 0 \qquad\qquad \text{for all } i = 1,2 \end{cases}$$

To maintain the biodiversity of species, it is natural to assume that all biomasses remain strictly positive, that is $X_j > 0$ for all $j = 1,2$; therefore $v > 0$.

As the scalar product of $(\lambda^i)_{i=1,2}$ and v is zero, so $\lambda^i = 0$ for all $i = 1,2$. So

$$\begin{cases} u^1 = 2pqAE^1 + pqAE^2 + c - pqX^* \\ u^2 = pqAE^1 + 2pqAE^2 + c - pqX^* \\ v = -AE^1 - AE^2 + X^* \\ < u^i, E^i > = 0 \qquad\qquad \text{for all } i = 1,2 \\ E^i, u^i, v \geqslant 0 \qquad\qquad \text{for all } i = 1,2 \end{cases}$$

thus

$$\begin{pmatrix} u^1 \\ u^2 \\ v \end{pmatrix} = \begin{bmatrix} 2pqA & pqA & A^T \\ pqA & 2pqA & 0 \\ -A & -A & 0 \end{bmatrix} \begin{pmatrix} E^1 \\ E^2 \\ 0 \end{pmatrix} + \begin{pmatrix} c - pqX^* \\ c - pqX^* \\ X^* \end{pmatrix} \qquad (12)$$

Let us denote by

$$z = \begin{pmatrix} E^1 \\ E^2 \\ 0 \end{pmatrix}, w = \begin{pmatrix} u^1 \\ u^2 \\ v \end{pmatrix}, M = \begin{bmatrix} 2pqA & pqA & A^T \\ pqA & 2pqA & 0 \\ -A & -A & 0 \end{bmatrix}, \ b = \begin{pmatrix} c - pqX^* \\ c - pqX^* \\ X^* \end{pmatrix}$$

then our problem is equivalent to the **L**inear **Complementarity P**roblem $LCP(M, b)$:

Find vectors $z, w \in \mathbb{R}^6$ such that $w = Mz + b \geqslant 0$, $z, w \geqslant 0$, $z^T w = 0$. It is easy to show that the matrix M is P-*matrice* and use the following result.

Theorem 1: $LCP(M, b)$ has a unique solution for every b if and only if M is a P-matrix.

Proof: See [15]-[18].

Therefore the linear complementarity problem $LCP(M, b)$ admits one and only one solution. This solution is given by

$$E^* = \frac{1}{3} A^{-1}(X^* - \frac{c}{pq}) \qquad (13)$$

where A^{-1} is the inverse of A, this matrix is given by

$$A^{-1} = \begin{bmatrix} \dfrac{r_1}{K_1 q_1} & \dfrac{c_{12}}{q_1} \\ \dfrac{c_{21}}{q_2} & \dfrac{r_2}{K_2 q_2} \end{bmatrix}$$

V. Numerical Simulations

We take as a case of study two marine species having the following characteristics:

$r_1 = 0,5$	$K_1 = 1000$	$q_1 = 0,1$	$c_{12} = 2.10^{-4}$	$c_1 = 0,05$	$p_1 = 40$
$r_2 = 0,3$	$K_2 = 700$	$q_2 = 0,02$	$c_{21} = 10^{-5}$	$c_2 = 0,10$	$p_2 = 60$

We'll see how changes in the price or the number of fishermen can affect the effort of catch, the level of captures and the profits of fishermen. As a first result we have (Table I): an increase in price leads to an increase in fishing effort and an increase in catch levels.

TABLE 1: An Increase in Price Leads to an Increase in Fishing Effort and Increase in Catch Levels

Price of the first specie	Price of the second specie	Total effort to catch the two species	Total captures of the two species
02,00	03,00	62,89	530,02
05,00	07,50	63,34	532,17
08,00	12,00	63,45	532,71
10,00	15,00	63,49	532,89
14,00	21,00	63,53	533,09
18,00	27,00	63,55	533,20
20,00	30,00	63,56	533,24
30,00	45,00	63,59	533,36
40,00	60,00	63,60	533,42

Now we will see the influence of the number of fishermen on the catch levels and on the profit (see Table II); to do so, we consider three situations:

In the first one we consider only one fisherman who catches the two marine species, to maximize the profit of this fisherman constrained by the conservation of the biodiversity of the two marine species, he must catch $569,92$, in this case his profit is equal to $7260,31$.

In the second one we consider two fishermen who catch the two marine species, to maximize the profit, each fisherman must catch $506,35$, in this case the profit of each fisherman is equal to $3226,80$, this situation reduces the catch of each fisherman by $88,85\%$ and reduces the profit of each fisherman by $44,44\%$.

In the third situation we consider ten fishermen who catch the two marine species, to maximize the profit, each fisherman must catch $188,24$, in this case the profit of each fisherman is equal to $240,01$, this situation reduces the catch of each fisherman by $33,03\%$ and reduces the profit of each fisherman by $3,31\%$.

So the three situations show that, when the number of fishermen is increasing, the catch and the profit of each fisherman are decreasing.

TABLE II: THE INFLUENCE OF NUMBER OF FISHERMEN ON THE CATCH AND PROFIT

	Catch/Fisherman	Profit/Fisherman
Situation n 1	569,92	7260,31
Situation n 2	506,35	3226,80
Situation n 3	188,24	240,01

On the contrary, since the number of fishermen is increasing, the total catch is increasing, but the total profit is decreasing.

Now we see that an increase in fishermen number leads to an increase in the total fishing effort and reduced the total profit as shown in Table III.

TABLE III: THE INFLUENCE OF NUMBER OF FISHERMEN ON THE TOTAL FISHING EFFORT AND TOTAL PROFIT

Fishermen number	Total fishing effort	Total Profit
01 fisherman	34,98	7260,31
02 fishermen	46,64	6453,61
03 fishermen	52,47	5445,23
05 fishermen	58,30	4033,50
10 fishermen	63,60	2400,10
15 fishermen	65,59	1701,63
20 fishermen	66,63	1317,06
25 fishermen	67,27	1074,01
30 fishermen	67,70	906,59
35 fishermen	68,02	784,29
40 fishermen	68,25	691,05
45 fishermen	68,44	617,61
50 fishermen	68,59	558,27

VI. CONCLUSION

We have calculated the fishing effort that maximizes the profit of each fisherman at biological equilibrium by using the Nash equilibrium problem. The existence of the steady states and its stability are studied using eigenvalue analysis. Finally, some numerical examples are given to illustrate the results.

In this work, we have considered that the prices of fish species are constants, we consider in a future work to define functions of providing long term, where price is no longer a constant but depends on the level of effort and biomass stock of each species remaining.

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Saida Bakht is a doctoral student at ENSA of Khouribga University Hassan 1st in Settat, Morocco. She obtained her Diploma of Advanced Higher Education from University Hassan II of Casablanca, Morocco Option Applied Mathematics in Engineering Science. Her research interests include: Study of diffusion reaction systems and equations: modeling, mathematical analysis and numerical resolution, LCP Problem of linear complementarity, bio-economic models, etc.

Youssef EL Foutayeni is a researcher at Hassan II University of Casablanca, Morocco. He teaches Mathematics in universities. He obtained doctorat from Cadi Ayyad University of Marrakech, Morocco. He also worked on School of Engineering and Innovation E2IM-Marrakech in Private University of Marrakech as a Director of Studies. His research interests include Linear Complementarity Problem LCP, Bio-economic models, Variational Inequalities VI, etc.

Nadia Idrissi Fatmi is a researcher at Hassan I University of Settat, Morocco. She teaches applied mathematics in universities. She obtained doctorat from Chouaib Eddoukali University of El Jadida, Morocco. Her research interests include study of diffusion reaction systems and equations: modeling, mathematical analysis and numerical resolution, study Bio-economic models, etc.