

Empirical Study of Markowitz Portfolio Theory and Model in the Selection of Optimal Portfolio in Shanghai Stock Exchange of China

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Abstract—In this paper, Markowitz's Portfolio Theory and Model are empirically studied under the condition that short selling is not allowed, taking Shanghai Stock Exchange A-share as the sample under the actual situation of China's securities market. This paper optimized the screening method of stock samples using two-level screening of correlation coefficient and coefficient of variation. By collecting the weekly return rate of sample stocks for three-year period and using this screening method, five stocks are included in the portfolio. By calculating the optimal weight of portfolio under the given expected return rate with these screened stocks will provide more accurate empirical results. After examination, the return rate of the portfolio is higher than that of Shanghai Composite Index in the latest month for most of the 22 effective trading days, which verifies the effectiveness of this method.

Index Terms—Markowitz portfolio theory, mean-variance model, optimal portfolio, efficient frontier, China's stock market, improved screening method.

I. INTRODUCTION

Since the global financial crisis in 2008, China's stock market has made great progress and carried out a series of market system reforms. In the past 10 years, the Shanghai Stock Index has a similar trend of "roller coaster", and the overall trend of domestic stock market presents long bear market and short bull market, which is not commensurate with China's economic development. Facing the reality of long bear market and short bull market in present domestic stock market, the return aspect is gradually weakened while the risk aspect is gradually highlighted. At this time, under the premise of minimizing risks and reducing losses to the minimum, it is crucial to obtain the maximum return to maintain or even increase the value of assets.

Markowitz's Portfolio Theory is exactly the solution to this problem. In recent years, many Chinese researchers have applied Markowitz Portfolio Theory and Mean-Variance Model to China's stock market for empirical research. Zeng Yinmiao, Zhang Jun, and Zhang Qing [1] built and solved portfolio model by calculating the average return rate and covariance matrix of securities in portfolio using data of 5 industry-leading stocks under the condition that short selling is not allowed. Li Qing [2] collected data of 10 stocks considering volume and calculated the optimal portfolio under two conditions of allowing and not allowing short selling, and analyzed the reasons for the deviation

between the theoretical prediction and the actual results. Liu Yaguang [3] graphed the effective frontier and verified the theory by collecting data of 52 industry-leading stocks and deduced the suitable investment crowd according to the risk of each stock. However, these methods of selecting stocks are not specific and enough, which only use the trading volume or industry representation of stocks as standard of measurement for example. Therefore, on the basis of previous studies, this paper uses improved Markowitz Mean-Variance Model based on the conditions of the China's stock market, taking Shanghai Stock Exchange A-share as sample, and optimizes the screening method of stock samples to verify the effectiveness of this method in order to obtain more accurate results for empirical test. This screening method is expected to improve the accuracy of experimental data results for the better empirical study of the Markowitz Portfolio Theory and Mean-Variance Model, and it will provide a new reference for Chinese investors' portfolio practice in terms of stock selection and portfolio returns and risks.

II. CONSTRUCTION OF MODEL

Harry Markowitz [4] published his seminal paper "Portfolio Selection" in 1952, which gave birth to the modern portfolio theory. It includes two important aspects: mean-variance model and efficient frontier model, which uses the expected rate of return and variance to respectively represent the investment value and investment risk of securities, and the efficient frontier of securities portfolio is derived finally. It is only rational for investors to construct a portfolio of securities on the efficient frontier. The basic principle revealed by Markowitz's portfolio theory is that investors always want to choose the portfolio with the highest return at given level of risk, or the portfolio with the lowest risk at given level of return.

Based on the assumption of Markowitz's portfolio theory that investors use the volatility of expected returns to estimate the risk of portfolio, the Mean-Variance Model under China's situation is constructed as follows:

$$\text{Objective function: } \min \sigma^2(r_p) = \sum \sum X_i X_j \text{Cov}(r_i, r_j) \quad (1)$$

where $\text{Cov}(r_i, r_j) = \rho_{ij} * \sigma_i * \sigma_j$, $r_p = \sum X_i r_i$, $\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$ ($-1 < \rho_{ij} < 1$).

Constraint: $1 = \sum X_i \geq 0$, since short selling is not allowed in mainland China [5].

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r_p stands for rate of return of the portfolio; r_i, r_j stand for the rate of return for the i th and j th stock, and X_i, X_j are their weights in the portfolio respectively; $\sigma^2(r_p)$ is the variance of the portfolio which stands for its variance; $Cov(r_i, r_j)$ is the covariance between the two stocks; ρ_{ij} is the correlation coefficient of the two stocks.

It can be seen from Equation (1) that the effect of risk diversification of securities portfolio mainly depends on the correlation coefficient between two securities, the standard deviation and weights of various securities. The effect of correlation coefficient on portfolio risk is greater than standard deviation. Only by choosing securities with small correlation coefficient can effectively diversify risk of investment, which can reduce the error between theory and practice.

III. METHODOLOGY

This paper optimizes the screening method of stock samples by performing two rounds of selection. The first-level screening includes the sector diversification, trading activity level, market cap, volume and regional representation. Then the second level of screening is conducted through the rank of coefficient of variation, and finally 5 stocks are included in the investment portfolio for empirical analysis. Due to Markowitz Portfolio Theory's rigorous hypothesis, dispersing the non-systematic risk by selecting stocks from different sectors and balancing risk and return using coefficient of variation in the stage of stock selection can be beneficial for reducing the error between theory and practice, which makes up for the deficiency in Li's research [2].

A. The Selection of Security Market

The reason for choosing China's securities market as the research object is that in recent years, with the rapid development of China's economic aggregate, the total market value of A Stock market has grown by leaps and

bounds, making it one of the important global emerging markets with the total market value ranking the second in the world [6]. Shanghai Stock Exchange has a large capital scale and stable profitability of mature enterprises, therefore, choosing it as the research object has strong typicality and representativeness.

B. Choice of Time Interval

The sample time period is selected from September 1, 2017 to August 31, 2020 with a total of 726 trading days, and samples of stock closing prices over the 158-week time period is collected. During this period, China's stock market has both continuous rising and continuous falling, so the empirical study of this time interval has a strong typical significance. It should be noted that although affected by COVID-19 outbreak at the beginning of 2020, the Shanghai Stock Index of the first quarter fell but by a small range, and also the China's stock market was generally stable during the epidemic compared with overseas markets, therefore, this external factor had little impact on the fluctuation of sample data.

C. Selection of Sample Stock to be Included in Portfolio

- 1) First-level screening: In this paper, the stocks in A Stock in Shanghai Security Exchange are taken as the sample set. To spread risk and reduce effect of non-systemic risk, it is necessary to reduce the correlation coefficient between stock by choosing different kinds of stocks. Therefore, based on Yahoo Finance's classification of 11 sectors: energy, basic material, financial services, consumer defensive, utilities, technology, real estate, consumer cyclical, health care, communication services, industry, 2 stocks in each sector are selected according to their trading activity level, market cap, volume and regional representation. 22 better-performing and representative stocks are selected. The IDs and symbols are shown in Table I.

TABLE I: IDS AND SYMBOLS OF 22 STOCKS (SOURCE: YAHOO FINANCE)

ID	Company Name	Sector	Symbol
1	Wintime Energy Co.,Ltd.	Energy	600157.SS
2	China Petroleum & Chemical Corporation	Energy	600028.SS
3	Inner Mongolia Junzheng Energy & Chemical Group Co.,Ltd.	Basic Material	601216.SS
4	Zijin Mining Group Company Limited	Basic Material	601899.SS
5	The Pacific Securities Co., Ltd	Financial Services	601099.SS
6	CITIC Securities Company Limited	Financial Services	600030.SS
7	Inner Mongolia Yili Industrial Group Co., Ltd.	Consumer Defensive	600887.SS
8	V V Food & Beverage Co.,Ltd	Consumer Defensive	600300.SS
9	GD Power Development Co.,Ltd	Utilities	600795.SS
10	China National Nuclear Power Co., Ltd.	Utilities	601985.SS
11	Sanan Optoelectronics Co., Ltd.	Technology	600703.SS
12	HNA Technology Co.,Ltd.	Technology	600751.SS
13	Poly Developments and Holdings Group Co., Ltd.	Real Estate	600048.SS
14	HNA Infrastructure Investment Group Co., LTD	Real Estate	600515.SS
15	Beiqi Foton Motor Co., Ltd.	Consumer Cyclical	600166.SS
16	Yonghui Superstores Co., Ltd.	Consumer Cyclical	601933.SS

17	Shanghai Fosun Pharmaceutical (Group) Co., Ltd.	Health Care	600196.SS
18	XIN JIANG READY HEALTH INDUSTRY Co., Ltd.	Health Care	600090.SS
19	China United Network Communications Limited	Communication Services	600050.SS
20	Dr. Peng Telecom & Media Group Co., Ltd.	Communication Services	600804.SS
21	China State Construction Engineering Corporation Limited	Industry	601668.SS
22	Hainan Airlines Holding Co., Ltd.	Industry	600221.SS

2) Second-level screening: In this paper, the weekly closing prices of the 22 individual stocks considering the reinvestment of cash dividends are extracted from Yahoo Finance for 158 weeks from September 1, 2017 to August 31, 2020. What is more, in order to solve the issue of continuous additivity of return rate, the logarithmic method is used to calculate the weekly return rate. Suppose the closing price of the i_{th} day ($i=0, 1, 2, \dots, n$) is S_i , then the continuously compounded rate of the i_{th} day can be expressed as:

$$R_i = \ln \frac{S_i}{S_{i-1}} \quad (2)$$

3) Then the average of weekly return rate can be shown as follows and in the equation $N=158$:

$$\bar{R}_i = \frac{\sum_{i=1}^N R_i}{N} \quad (3)$$

$$\text{Coefficient of variation} = \frac{\sigma}{\bar{R}_i} \quad (4)$$

4) Although a wide variety of stocks should be selected as far as possible to spread the risk when building the model, it is not that more stocks included in portfolio leads to a better outcome in spreading risk. According to Fama [7], increasing the number of assets from four to five will maximize the amount of risk reduction in the investment portfolio. Therefore, this paper includes five stocks with positive and lower coefficient of variation in portfolio. Lower correlation coefficient is consistent with the principle of dispersing non-systematic risk in step 3.3.1.

D. Data Processing

1) Calculate variance covariance matrix of the five stocks

using Excel.

2) Substitute data into the improved $M-V$ model based on actual situation in China:

$$\min \sigma^2(r_p) = \sum \sum X_i X_j \text{Cov}(r_i, r_j) \quad (5)$$

$$r_p = \sum X_i r_i \quad (6)$$

3) Constraint (no short selling):

$$1 = \sum X_i \geq 0 \quad (7)$$

4) Use the Solver tool of Excel to get five set of efficient portfolios under five assignments of expected return rate of investment portfolio. Then plot part of efficient frontier using the five portfolios.

5) Verify the effectiveness of this screening method: Use a graph to verify the effectiveness of this screening method by comparing the daily return rate of the portfolio in September 2020 using the weight calculated above that under a certain expected return rate and the daily return rate of Shanghai Composite Index in September.

IV. RESULTS

Table II shows the realigned results in ascending order based on coefficient of variation after calculating weekly average return rate and its variance, standard deviation and coefficient of variation for the 22 stocks. The five stocks with IDs 7,6,4,1,13 have lower and positive coefficients of variation, reflecting their better performance in the balance of risk and return.

TABLE II: MEAN, VARIANCE, STANDARD DEVIATION AND COEFFICIENT OF CORRELATION OF THE 22 STOCKS' WEEKLY RETURN RATE
SOURCE: YAHOO FINANCE, OWN CALCULATION

ID	Mean	Variance	Standard deviation	Coefficient of Variation
5	-0.0140%	0.0035	0.0595	-423.8576
8	-0.0641%	0.0022	0.0470	-73.3896
15	-0.0677%	0.0024	0.0491	-72.4961
2	-0.1153%	0.0011	0.0337	-29.1846
21	-0.1364%	0.0012	0.0345	-25.2848
14	-0.3574%	0.0065	0.0807	-22.5924
20	-0.3750%	0.0048	0.0695	-18.5260
12	-0.3464%	0.0038	0.0616	-17.7818
19	-0.2565%	0.0016	0.0397	-15.4825
1	-0.5904%	0.0044	0.0664	-11.2449
10	-0.2764%	0.0007	0.0269	-9.7221

22	-0.4438%	0.0017	0.0415	-9.3560
9	-0.3533%	0.0006	0.0249	-7.0395
18	-0.9252%	0.0024	0.0491	-5.3117
7	0.4187%	0.0020	0.0443	10.5893
6	0.3876%	0.0023	0.0483	12.4708
4	0.4130%	0.0028	0.0528	12.7827
17	0.4158%	0.0036	0.0598	14.3889
13	0.3691%	0.0030	0.0545	14.7699
3	0.4713%	0.0060	0.0776	16.4725
16	0.1809%	0.0020	0.0446	24.6853
11	0.2369%	0.0050	0.0704	29.7344

The results of correlation coefficient verification for the above five stocks with small and positive coefficient of variation are shown in Table III. According to Table III, correlation coefficient of the selected five stocks generally

low, which conforms to the principle of spread the risk of non-system in the former step of selecting stocks. Therefore, these five stocks are included in the investment portfolio.

TABLE III: CORRELATIONS OF THE FIVE STOCKS

ID	4	6	7	13	17
4	1.000	0.506	0.339	0.423	0.290
6	0.506	1.000	0.331	0.511	0.304
7	0.339	0.331	1.000	0.236	0.400
13	0.423	0.511	0.236	1.000	0.113
17	0.290	0.304	0.400	0.113	1.000

TABLE IV: VARIANCE COVARIANCE MATRIX OF THE FIVE STOCKS

ID	4	6	7	13	17
4	0.002787	0.00129	0.000794	0.001217	0.000917
6	0.00129	0.002337	0.00071	0.001348	0.00088
7	0.000794	0.00071	0.001966	0.000571	0.001061
13	0.001217	0.001348	0.000571	0.002972	0.00037
17	0.000917	0.00088	0.001061	0.00037	0.003579

Table IV shows the variance covariance matrix of these five stocks calculated using Excel. After substituting above results into the improved M-V model and through five assignments 0.4100%, 0.4130%, 0.4150%, 0.4180%,

0.4187% of expected return rate of investment portfolio, results of five efficient portfolios including the selected five stocks shown in Table V.

TABLE V: FIVE OPTIMAL PORTFOLIOS

	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5
Weight for stock 4	20.25%	23.68%	26.13%	6.29%	0.00%
Weight for stock 6	11.82%	9.28%	5.66%	0.00%	0.00%
Weight for stock 7	46.48%	49.99%	52.68%	81.76%	99.70%
Weight for stock 13	6.96%	2.08%	0.00%	0.00%	0.00%
Weight for stock 17	14.49%	14.97%	15.53%	11.95%	0.30%
Portfolio Variance	0.00128	0.00134	0.00139	0.00168	0.00196
Port Standard deviation	0.03575	0.03659	0.03731	0.04097	0.04428
Portfolio expected return	0.4100%	0.4130%	0.4150%	0.4180%	0.4187%

Fig. 1 plots part of efficient frontier using the five portfolios. It shows that as the expected return rates of the optimized portfolio increases, the risk increases, and the increasing rate of risk is higher than the increasing rate of return.

Fig. 2 shows the comparison of return rate between M-V model and Shanghai Composite Index in September 2020.

When expected return rate of this portfolio is 0.4130%, the return rate of portfolio is higher than that of Shanghai Composite Index in September 2020 for most of the 22 effective trading days. This indicates that the stock portfolio obtained by this screening method is effective in the long run.

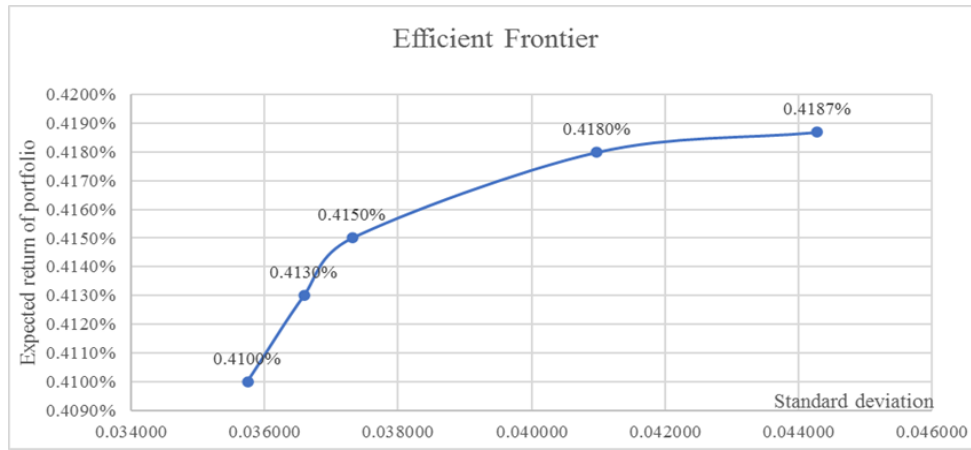


Fig. 1. Efficient frontier.

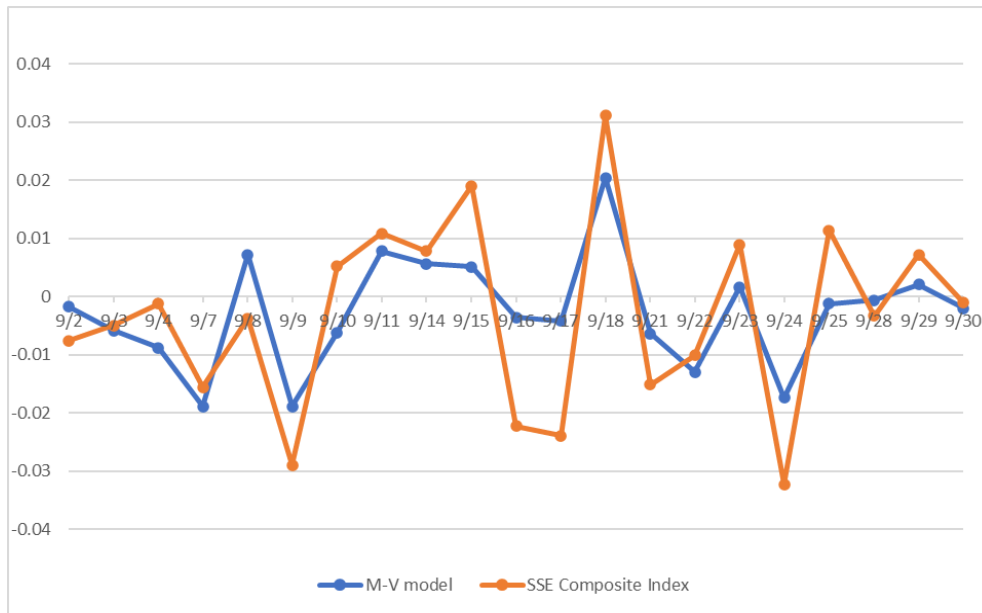


Fig. 2. Comparison of yield between M-V portfolio model and Shanghai Stock Exchange in September 2020.

V. DISCUSSION

The information drawn from Fig. 1 and Table V can provide a good theoretical basis for investment practice.

Firstly, according to Fig. 1, by assigning different values of expected return rates to different portfolios under M-V model and solving it, the optimal investment scheme corresponding to the minimum risk (standard deviation) at different return rates can be obtained, and the efficient frontier indicates that as the expected return rates of the optimized portfolio increases, the risk increases, and the increasing rate of risk is higher than the increasing rate of return. Therefore, a higher expectation of rate of return means taking a higher risk.

Secondly, it can be seen from Table II and Table V that among the optimal portfolios from 1 to 5, stocks with smaller variance and coefficient of variation account for the largest weight in the portfolio. For example, stock 7 has smallest variance and coefficient of variation, respectively 0.001966, 10.5893, comparing to other four risky assets in portfolio. With the increase of weekly return rate of each portfolio, it always has the largest weight. This phenomenon can be understood as assuming that there are two kinds of asset investment will obtain the same return, people tend to

choose the less risky assets to invest, which reflects the characteristics of a rational investor to seek advantages and avoid disadvantages.

Thirdly, as the expected return rate increases from 0.4100% to 0.4187%, it can be seen from Table V that the number of stocks involved in the optimal portfolio decreases from 5 to 2. This result is consistent with result of Markowitz's modern portfolio theory: Markowitz's efficient portfolios contain a small number of securities and is concentrated, rather than managing resources evenly as equal-weight portfolios do, because it focuses on reducing the correlation between various returns and excluding some securities with low returns and high risks. Therefore, when investors pursue higher returns, the number of securities in the effective portfolio is small and concentrated.

VI. CONCLUSION

Above results show that the Markowitz portfolio theory has significant application value in China's stock market. Based on the two-level selection of high-quality A-Shares in Shanghai Stock Exchange which effectively reduces non-systemic risks in the investment portfolio by choosing stocks with lower correlation coefficient and balances the

return and risk by ranking coefficient of variation, 5 stocks are included in the portfolio and efficient frontier is graphed using five optimal portfolios under the given expected return rate. It shows that there is a positive correlation between return rate and risk; People tend to choose assets with less risk under the same return rate; Markowitz's efficient portfolios is concentrated with small number of securities. The effectiveness of the screening method is also verified in September 2020's investment practice that the actual return rate of the portfolio is above SSE Index, which shows that Markowitz portfolio theory can be used to obtain a better portfolio than the average performance of the whole stock market in China's current stock market. This method also provides more accurate results for empirical analysis, which makes up for the deficiency in previous research.

At the same time, there exist some limitations of the application of Markowitz portfolio theory in China. First, Markowitz's portfolio theory is based on the efficient markets hypothesis [7], but it is ideal since market failure is inevitable in reality. Second, Markowitz's model does not consider transaction costs during portfolio investment, which cannot be ignored in portfolio management practice, and inefficient portfolio may be caused. Therefore, due to the limitations of the theory itself, investors should not blindly copy the theory to guide their investment decisions. In practice, they should also make appropriate adjustments according to the actual situation. Only in this way can they avoid unnecessary losses.

CONFLICT OF INTEREST

The author declares no conflict of interest.

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