Generalized Revenue Management Model for Small Hotels

Jose-Mario Zarate-Carbajal, Riemann Ruiz-Cruz, and Juan-Diego Sanchez-Torres

***Abstract*—Revenue management models have evolved through time; the benefits and implementation complexity have increased. Large hotel chains have successfully implemented revenue management models; in contrast, small hotels lag in adoption. We propose a simplified revenue management model more suitable to be adopted by small hotels. We simulate customer attendance using parameters from a travel agency. We adapt two popular revenue management from airlines to a small hotel system, and we describe a method to find the optimal parameters for each model. We compare the revenue's performance in the system, and we prove a revenue increase by implementing our models. Finally, we propose a simplification based on the single capacity allocation model and suggest a set of business rules for implementation.**

***Index Terms*—Revenue management, simulation modeling, numerical optimization, hotel management.**

I. Introduction

A Revenue Management (RM) model improves the income of a system. Their basic concept is to find the right price for the right customer at the right time. RM models have been implemented in highly competitive sectors a defined customer segmentation, at least with two customer groups. One pays less by booking the product or service in advance, and the other pays more by booking with limited time.

The literature review for RM in the airline industry is abundant, and models have evolved to incorporate more factors. E. Shlifer and Y. J. T. S. Vardi [1] proposed a full airline booking policy for companies. R. W. Simpson [2] proposed a rejection policy to accept bookings only if the current booking revenue is higher than the expected revenue for future bookings. S. L. Brumelle and J. I. McGill [3] include the overbooking factor in the RM models for Air Fare. A. Gosavi *et al.* [4] and A. M. Fouad *et al.* [5] developed further research on overbooking and cancellation policies. B. Vinod [6] made an analysis of the revenue management system by origin and destination city. J. L. Higle [7] extended this view by adding a stochastic programming approach for the same problem.

In contrast, the research of RM models in the hotel industry is relatively new. S. E. Kimes [8] initially proposed the idea of yield management for hotels and restaurants without a sense of dynamic pricing. G. R. Bitran and S. V. Mondschein [9] consider the management of dynamic prices when a customer stays multiple days in a hotel [10], R. D. Badinelli [11] suggest dynamic policy management for hotels with low capacity. J. Wirtz *et al.* [12] explores the fairness. M. Müller-Bungart [13] dedicates a section in his book to consolidate the main RM, [14] revisit the RM [15] make a study case where he reports revenues improvement by applying RM models in hotels between 2% to 5%. S. E. Kimes [16] provides an overview of the future of RM in hotels, suggesting that the whole industry has to implement them in the same way as the airline industry did.

The increase of computational power has made popular meta-heuristic techniques in optimization. A. Nuno *et al.* [17] uses machine learning techniques to predict the cancellation of bookings. K. Subulan *et al.* [18] revisit RM models and propose a self-adjusted optimization function. W. H. Lieberman [19] considers robotics and actual pricing to be an opportunity area for optimizing RM models. E. Hertzfeld [20] highlights the use of Expanded AI in the models. The trend on Revenue Management models is moving to optimization self-adjusting systems focusing on large hotel chains. R. Thomas *et al.* [21] highlights the importance of small participants in the sector, and they recognize the challenge of having a definition of small hotels. The most general description of hotels is by the number of rooms. We focus on models suitable for small hotels with less than 30 rooms. In this paper, we do a business financial study to simulate a hotel booking system and implement traditional Airline RM models: nested capacity single/ double allocation [22] and resource bid Price [2]. We propose a simplification suitable to be implemented in small hotels.

II. Nested Capacity Allocation

The nested capacity allocations, also known as the booking limit, were first proposed by K. Littlewood [22], and it was thoroughly extended by P. Belobaba [23]. The booking limit approach consists of reserving a specific amount of space for each type of customer. The model defines an optimal combination between business and tourist customers to maximize the revenue in the system for each capacity. Under this approach, a fixed number of spaces are allocated for business customers out of the total capacity. A trivial example is when we know business customers arrive often and the capacity is one; in this case, the system allocates the available space for them; in other words, bookings in advance are not allowed. The single allocation considers only one constraint; the double allocation considers two constraints. For example, the one restriction model allocates rooms by a certain customer type; the two restrictions model allocates rooms by a certain customer type and by a specific weekday of arrival. The challenge of nested capacity allocation models is finding the right proportion of allocated spaces for each customer type. We refer to the process to find this proportion as model calibration.

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III. Resource Bid Prices

The resource bid pricing model was proposed by R. W. Simpson [2] and extended by E. L. Williamson [24]. This model consists of constructing a price matrix based on customers' total arrivals from both classes (business and tourist). The model requires the expected price per night for bookings. Using this value, we will create an expected revenue function. The expected revenue function is also referred to as bid prices. This expectation is a combination of the rejected parameter and the probability distribution function that includes a capacity constraint. We use the bid price function to compare the revenue of the current booking against the expected revenue. A new customer is accepted if the current booking price exceeds the sum of the bid price per night.

The capacity constraint and the hotel's current occupancy play an essential role in the bid price function. If the capacity is close to being reached, customers from the tourist class are rejected, expecting a business class customer to arrive. If there are two spaces available, at least two business class bookings should come to make profitable the decision to reject the current booking. If there are three spaces available, at least three business class customers should arrive. It is most likely that one business customer arrives than two, it is more likely that two business customers arrive than three, and so on. The probability of arrival of a business customer is a function of the number of spaces available.

In the model, the business customer follows a Poisson distribution. The probability that at least X business customers arrive in the system is given by equation (1):

$$P(X) = 1 \sum_{i=0}^{X} \left[\exp(-D) \left(\frac{D^{X_i}}{X_i!} \right) \right] \tag{1}$$

where X is the spaces remaining in the system, D is the duration of nights of the customer at the hotel. The main assumptions of the model are: *the value of the money through the time is not considered, the hotel does not accept cancellations; therefore, the system receives full income once the booking is accepted, when a booking is rejected, there is not a possibility to return in another period, the income of rejected bookings is lost permanently*.

The system rejects the booking if the current booking revenue is lower than the expected potential revenue of a new booking. The potential revenue is calculated by equation (2)

$$PR = P(X) \left(\overline{p(B)} \right) \tag{2}$$

where PR: potential revenue, $P(X)$: probability that at least X business customers arrive in the system, and $\overline{p(B)}$: average prices per night estimated from historical data or simulated values. This equation accepts customers only if their revenue is higher than the potential revenue per night of news customers. Sometimes, the expected price may not reflect the reality of the pricing; we can apply some techniques to improve this factor. In this paper, we propose to set $\overline{p(B)}$ as the maximum price per night (maximum rate of high paying customer $p(B)^+$) and to add a rejection parameter. By setting $p(B)^+$ as a fixed value, we focused only on finding the optimal $\hat{\theta}$ parameter for the system. We modify equation (2) as follows:

$$PR = P(X)(p(B)^+)(\theta) \tag{3}$$

where PR: potential revenue, $P(X)$: the probability that at least X business customers arrive in the system, and $p(B)^+$: maximum price per night that the system allows, and θ rejection parameter.

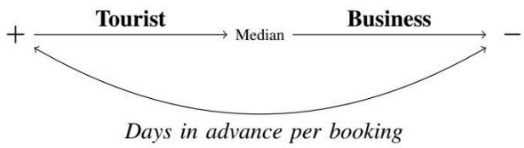

Fig. 1. The classification of customers is based on the median of the days in advance of the bookings. A tourist customer registers a booking more days in advance than a business customer.

IV. Methodology

Data availability in the sector is highly restricted; the booking data is considered part of the business know-how, and it is not shared at a granular level. Most of the published papers related to RM modeling in hotels rely on Monte Carlo simulations. The demand behavior in hotels is generally divided into seasons, *summer season: from 31/Mar to 30/Sep and winter season: from 01/Oct to 31/Mar*. We managed to get high-level parameters from a travel agency; the high-level parameters explain the customer influx (accepted and rejected customers) of a small hotel in a US city during the summer. Demand in summer overpasses the capacity; therefore, implementing a revenue management system takes more relevance. We focus our research on small hotels with 5 to 30 rooms

Small hotels divide the customer into two types: business and tourist, commonly differentiated by the median of the days of booking in advance. Fig. 1 shows a visualization of the division of business and tourist customer by the median. A tourist customer registers a booking with more days in advance than a business customer; we have the rate of arrival of the customer by weekday, the average and variance of days of booking in advance, and the number of nights per customer type. The summary of the agency's parameters is in Table I and Fig. 2 shows the arrival rate by each class of customer over the different days of the week. Business customers arrive mostly on Sun, Mon, Tue, and Wed (Group 1). Tourist arrive mostly on Thu, Fri, Sat (Group 2).

TABLE I: Demand Parameters from Travel Agency

Advance	All	Business	Tourist
Mean μ_a	19.56	6.18	32
Std σ_a	19.61	3.87	20.16
Median	15	6	25
Duration	**All**	**Business**	**Tourist**
Mean μ_d	3.94	3.53	4.33
Std σ_d	3.12	2.96	3.21
Arrivals p/day	**All**	**Business**	**Tourist**
Sun	5.85	4.23	1.62
Mon	6.88	6.69	0.19
Tue	3.42	3.19	0.23
Wed	2.81	1.77	1.04
Thu	6.77	0.69	6.08
Fri	6.96	0.19	6.77
Sat	2.69	0.27	2.42

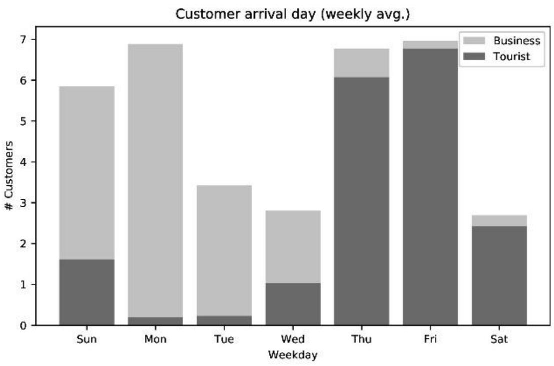

Fig. 2. There is clear trend of the arrival day for business and tourist customers. Business customers arrive mostly on Sun, Mon, Tue, and Wed (Group 1). Tourist arrive mostly on Thu, Fri, Sat (Group 2).

We use agency parameters to simulate client attendance. We assume Poisson distributions for arrivals and Normal distributions for days in advance, and the number of nights. The parameters for the simulation are in Table II.

Firstly, we simulate customer demand for 365 days (simulation period); for the analysis, we consider only 184 days (summer period: from 31/Mar - 30/Sep) to match agency parameters. We study the simulated customer attendance under two capacity scenarios: 10 and 20 rooms. Secondly, we propose a method to calibrate the capacity allocation (single-double) and the resource bid prices model. Thirdly, we compare the RM models using the optimal parameters to the No Model (NM) implementation scenario, and we also extend this comparison to other author's works. Finally, we select the RM model that is most suitable to be implemented in a small hotel. We increase the number of simulations to propose a generalized model; we test our proposal under different conditions and suggest a set of business rules.

TABLE II: PARAMETERS FOR SIMULATION

Parameter	Business customer	Tourist customer
B_n	$\sim$ N$(6.18,\ 3.87^2)$	$\sim N(32, 20.16^2)$
D_n	$\sim N(3.53, 2.96^2)$	$\sim N(4.33, 3.21^2)$
λ_{Sun}	$\sim P(4.23)$	$\sim P(1.62)$
λ_{Mon}	$\sim P(6.69)$	$\sim P(0.19)$
λ_{Tue}	$\sim P(3.19)$	$\sim P(0.23)$
λ_{Wed}	$\sim P(1.77)$	$\sim P(1.04)$
λ_{Thu}	$\sim P(0.69)$	$\sim P(6.08)$
λ_{Fri}	$\sim P(0.19)$	$\sim P(6.77)$
λ_{Sat}	$\sim$ P(0.27)	$\sim P(2.42)$

A. Client Attendance Simulation

We use a Monte Carlo simulation to replicate a hotel system's behavior similar to P. Jonhson' result [25]. Each customer booking has three attributes: booking time (t_b), arrival time (t_a), and departure time (t_d). The simulation function for a n number of customers C_n in a given day t_k is model by 4,

$$\begin{matrix} C_n = \\ t_a = \\ t_b = \\ t_d = \end{matrix} \begin{vmatrix} C_1 & C_2 & \dots & C_n \\ t_k + A_1 & t_k + A_2 & \dots & t_k + A_n \\ t_{a_{C_1}} - B_1 & t_{a_{C_2}} - B_2 & \dots & t_{a_{C_n}} - B_n \\ t_{a_{C_1}} + D_1 & t_{a_{C_2}} + D_2 & \dots & t_{a_{C_n}} + D_n \end{vmatrix} \quad (4)$$

where C_n: customer booking i of n customers (C_n simulates $\sim P(\lambda)$ times), t_k is a given day of the simulation (day 1, day 2,..., day 365) of K simulated days, t_a: day of arrival, t_b: time of booking, t_d : time of departure, $A_n \sim U(0,0.499)$, $B_n \sim N(\mu_a, \sigma_{wd_a})$, $D_n \sim N(\mu_d, \sigma_d)$.

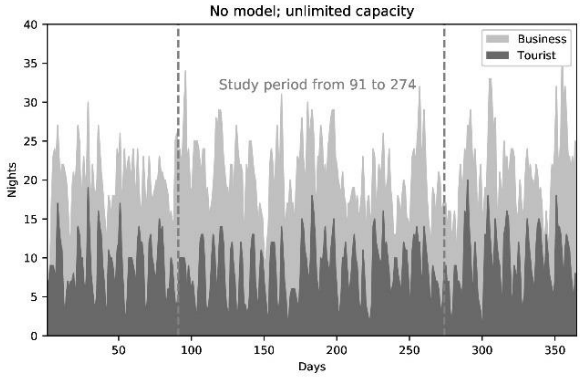

Fig. 3. This figure shows the customer occupancy of each day for the simulation by customer type before any capacity constraint. We selected only the summertime (from 91 to 274) for the study.

For example, to simulate the influx and parameters of business customers that arrive on Saturday, February 24th, 2018. t_k=55 (k day of the year), the number of customers C_n simulates $\sim P(0.27)$ times, $A_n \sim U(0,0.499)$, $B_n \sim N(6.18, 3.87^2)$, $D_n \sim N(3.53, 2.96^2)$. The system works with round numbers; this means that the customers pay a full night whether they arrive very late or leave very soon in the morning. The function to determine whether if the customer is in the hotel or not (pays a full night) is

$$(C_i) = \begin{cases} 1 & if \quad t_k \le t_a < t_{k+1} \\ 1 & if \quad t_k < t_d < t_{k+1} \text{ and } t_a < t_d \\ 1 & if \quad t_a < t_k \text{ and } t_d > t_{k+1} \\ 0 & if \quad \text{Otherwise} \end{cases} \quad (5)$$

where tk is the actual day of the simulation, and the other variables are the same as in equation (4).

The price for a given customer (C_i) depends on the number of days in advance and the number of nights of each booking. The more days the customer books in advance and the more days a customer stays, the lower their price per night. The price parameters vary by season, city, and hotel size; the travel agency provides an insight into how the price is calculated for small hotels during summer. In our systems, the price per night is determined by equation

$$p(C_i) = \alpha + \frac{2}{D}\beta + \frac{7}{A}\Gamma \quad (6)$$

where $p(C_i)$: price per night, D: duration of nights of the customer at the hotel $(t_d - t_a)$, A: number of days in advance of the booking $(t_a - t_b)$, α, β, Γ: positive constant with the following values: $\alpha = 60$, $\beta = 30$, $\Gamma = 30$. Value 2 relates to the limit where the hotel is willing to start offering a discount for a longer stay; value 7 is associated with the point where the hotel is willing to offer a discount for booking in advance. The maximum price per night is \$330.00 when the customer stays a single night $(D = 1)$ and makes the booking the minimum days in advance $(A = 1)$.

The simulation is represented in Fig. 3. The booking simulation period starts from 1 to 365 days (day 1, day 2, day t_k, ..., day 365); we use parameters from ~ Table II. We simulate arrivals for business and tourist customers; we merge both data sets and sort them by booking time. Despite having the two simulation data sets from business and tourist customers, their classification is based on the median of the days in advance of the bookings, the same as a small hotel normally does. The customers can book any room within the simulation period; the analysis period covers only the summer season, from 91 to 274 (184 days).

V. Results

We simulated customers arrivals for 365 days for two classes of customers (business and tourist) using parameters from Table II. The tourist customers make bookings in advance; for this reason, during the initial days of the simulation, they do not arrive at the hotel; tourists also make bookings in the last days, they book to arrive after the period of the simulation (day 365); this effect is known as "simulation tails". The tails in the simulation affect the revenue calculation. We focused our analysis on the summer period (from day 91 to 274), where we do not have "simulation tails." The hotel occupancy by type of customer from the simulation is shown in Fig. 3. The maximum number of rooms required to satisfy the demand is 34; the average occupancy for business customers is 8.2, and 12.2 for tourist customers.

We studied the behavior of the bookings under two capacity constraints: 10 and 20 rooms. In a No Model (NM) scenario, the hotel accepts the customers as they make the bookings. The results of the simulation under NM implementation are presented in Table III. Tourist customers get more rooms because they book before business customers; the fewer rooms available, the more % of tourist customers in the hotel.

TABLE III: Results of Simulation Adding Capacity Constraint

No model (NM)	10 rooms	20 rooms	No restriction
Rev ($k)	$144.60	$288.40	$385.10
# Cust.	464	813	958
%Bus	31.50%	41.70%	47.40%
%Tou	68.50%	58.30%	52.60%

A. Optimization

An RM model finds the right balance between business and tourist customers to maximize revenue. We use the simulated data set to find the optimal parameters for both models: nested capacity single/double Allocation and resource bid pricing model. The single allocation model is calibrated by increasing the percentage of allocated rooms for business customers out of the total capacity and comparing each scenario's revenue; we explore all possible combinations between business and tourist customers. For example, for a capacity of 10 rooms, we recreate the income for the hotel when zero rooms are allocated for business customers and ten rooms for tourist customers [0,10], then one room for business customers and nine rooms for tourists [1,9], [2,8], [3,7], ..., [10,0]. The optimal number to allocate for business customers is when we get the highest revenue. The results of the calibration process are in Fig. 4; the revenue reaches a peak and then decreases when we have more rooms allocated for business customers. For the single allocation model, the optimal parameter for capacity 20 rooms is 35% business customers (seven business, thirteen tourists [7,13]); for capacity 10 rooms is 40% business customers (four business, six tourists [4, 6]).

The double allocation model adds an additional restriction for tourist customers. We allocate more rooms for the business customers by blocking bookings in advance that arrive on the weekdays where business customers most commonly arrive, refer to Fig. 2. For a given number of rooms allocated for business customers, we evaluate the combination for tourist customers in the weekday group two. For example, for a capacity of 10 rooms, we review the revenue when 0 rooms are allocated for business customer and 10 rooms for tourist customers to arrive on Sundays, Mondays, Tuesdays or Wednesdays (weekday group 1) and 0 rooms for tourist customer to arrives on (weekday group 2) [0,10,0], then for 0 rooms for business customers and 9 rooms for tourists customers weekday group 1, and 1 room for tourist weekday group 2, then [0,9,1], [0,8,2], [0,7,3], ..., [1,9,0], [1,8,1], [1,7,2], ..., [10,0,0]. The optimal number to allocate for the business customers and tourist's weekday group is when we get the highest revenue. The dots in Fig. 4 shows the calibration process for the double allocation model; the optimal parameters for capacity 20 rooms is 35% business customers (seven business, eleven tourist group 1, two tourist group 2 [7,11,2]; for capacity 10 rooms is 40% business customers (four business, five tourist group 1, one tourist group 2 [4,5,1].

On the other hand, in the bid pricing model, we find the optimal rejection parameter (θ) from equation (3). $p(B)^+$ is calculated from equation (6) considering a customer who books with no days in advance for a single night ($330). The optimal $\hat{\theta}$ is a value between 0 and 1; when $\theta = 1$, the system rejects those customers who are not paying the maximum rate, we over reject customers. In contrast, when $\theta = 0$, the system does not reject any customer; this scenario is equivalent of not implementing any model (NM). The bid pricing model accepts or rejects a customer considering the probability that new costumers that pay more arrive, regardless of the customer type (business or tourist). We find the $\hat{\theta}$ that maximized the revenue, and the process is in Fig. 5. The $\hat{\theta}$ for capacity 20 rooms is 0.3, and for capacity 10 rooms is 0.42.

B. Models Results

We studied the single allocation, the double allocation, and the bid pricing models using the optimal parameters and comparing them to letting the customers arrive as they book; this is equivalent as No Model (NM) implementation. Fig. 6 shows the customer occupancy per day in the hotel by type of customer for a capacity constrain of 10 rooms; the charts shows that under NM implementation, the hotel accepts more tourist customers; under the single and double tourist customers are limited to a given number of rooms; under the resource bid pricing the business customers occupancy varies per day. The behavior is similar for a capacity constraint of 20 rooms.

All the Revenue Models (RM) in this study increased revenue compared to NM implementation. The numerical results are presented in Table IV. The single and double allocation models have similar results for both capacities. In the scenario where the capacity of the hotel is 10 rooms, the best model is the resource bid pricing, it increased the revenue 8.15%, almost twice as the single (4.34%) and the double allocation (4.07%); results in the capacity of 10 contrast to the capacity 20 scenario; in this scenario, the bid pricing was the worse, increasing the revenue only by 0.48%, the best model is the double allocation with 1.67% increase, follow by the single allocation with a 1.59% increase. The results show that the impact of the revenue management model is higher when the capacity is lower.

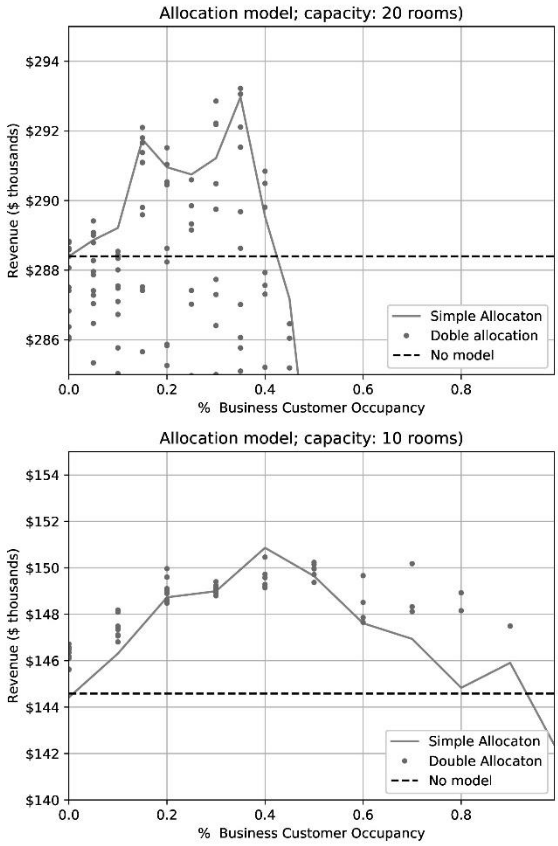

Fig. 4. This figure shows how the revenue behaves as the % of rooms allocated for business customers increase. The solid line shows the result for the single allocation model, the optimal parameters for capacity 20 rooms: 35% business customers (seven business, thirteen tourists); for capacity 10 rooms: 40% business customers (four business, six tourists). The dots show the results for the double allocation model, and we add the weekday condition; the optimal parameters for capacity 20 rooms: 35% business customers (seven business, eleven tourists group 1, two tourists group 2; for capacity 10 rooms: 40% business customers (four business, five tourists group 1, one tourist group 2.

TABLE IV: RM MODELS W/ OPTIMAL PARAMETERS VS NO MODEL (NM)

10 rooms	NM	Single	Double	Bid Pricing
Rev ($k)	$144.60	$150.90	$150.50	$156.40
% vs NM	0%	4.34%	4.07%	8.15%
# Cust.	464	453	450	546
%Bus	31.50%	51.90%	53.60%	48.40%
%Tou	68.50%	48.10%	46.40%	51.60%
20 rooms	NM	Single	Double	Bid Pricing
Rev ($k)	$288.40	$292.90	$293.20	$289.80
% vs NM	0%	1.59%	1.67%	0.48%
# Cust.	813	796	795	826

C. Results Comparison to Other Authors

We compared the results of this paper to other authors who used similar RM models in the hotel sector. The data set used by each of the authors is not available, we cannot perform a direct comparison, the revenue amounts, demand parameters, pricing, and capacity are different in each work. The comparison has no intention to suggest that some work is better than the other, Table V is presented for reference purposes only, we used the percentage of improvement in revenue.

The % average improvement across all the papers is 9.9%, although we cannot make a direct comparison, this improvement seems to be equivalent to the benefits reported in the Airlines (10% improvement year over year), [26].

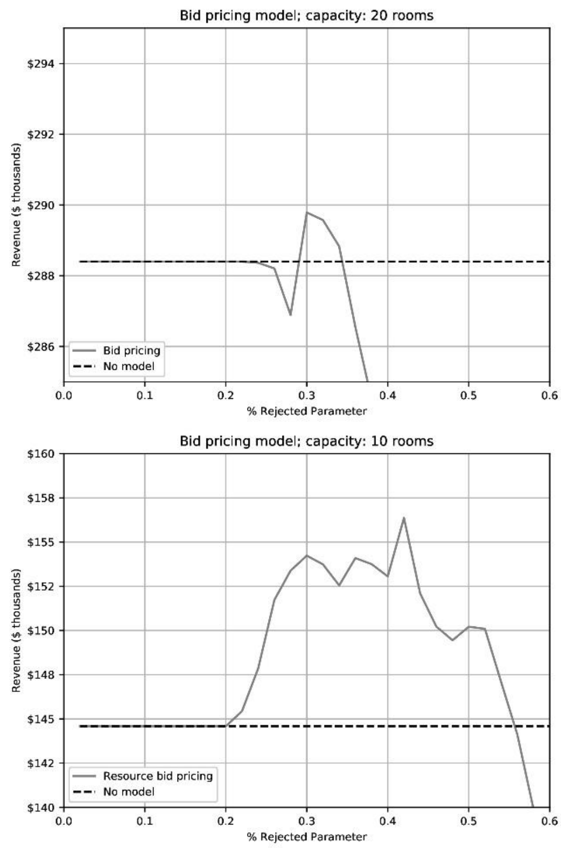

Fig. 5. This figure shows how the hotel's revenue behaves in the resource bid pricing model as the rejection parameter increases. If $\theta = 1$, the system rejects those customers who are not paying the maximum rate, we over reject customers; if $\theta = 0$, the system does not reject any customer at all; The $\hat{\theta}$ for capacity 20 rooms: 0.3 and form capacity 10 rooms: 0.42.

TABLE V: COMPARISONS TO OTHER AUTHORS

Autor	% improvement	Model
This paper	1.59%-4.34%	Single allocation - optimization
This Paper	1.67%-4.07%	Double allocation - optimization
This Paper	0.48%-8.15%	Bid pricing - optimization
(Kimes, 2010) – [15]	2%-5%	Price - segmentation
(Baker & Collier, 2003) – [26]	~10%	Heuristic - Price setting
(Pimentel et al., 2019) – [27]	~20.2%	Price setting - overbooking
(Nuno et al., 2017) – [17]	11.8%- 26.4%	Machine learning - overbooking

D. Proposal

The most recent RM models for hotels concentrate on heuristic and machine learning techniques; although these new models, according to the authors, improve the system's revenue, they become more complex, and their implementation requires hiring costly resources; these models seem to be restricted only to large hotel chains. This paper focuses on small hotels, and we propose the most feasible model to be implemented in this sector. Instead of moving in the direction of machine learning and heuristic methods (more complex models), we move to model simplification; the single allocation model provides a simple rule to accept or reject customers that most likely any small hotel can implement.

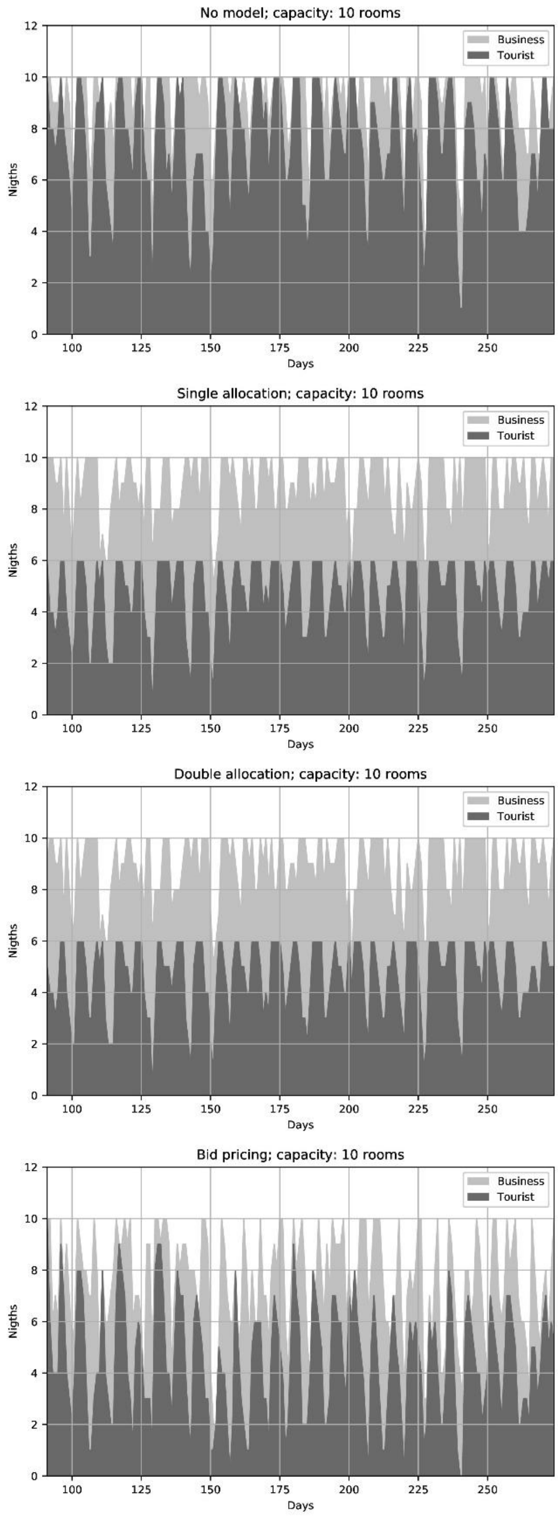

Fig. 6. Using a capacity constraint of 10 rooms, under NM implementation, the hotel accepts more tourist customers; under the single and double allocation model, the tourist customers are limited to a given number of rooms; under the bid pricing model, the business customers occupancy varies per day.

The data availability in small hotels is related to fiscal obligations, and we consider any small hotel has at least ten years of data; for this reason, we extend the client attendance Monte Carlo simulation to ten simulations, this equivalent to getting data from a hotel for the last ten summers.

We find the optimal mix for the optimal % of business customers using the single allocation model. We replicate the optimization process for all capacities between 5 and 25 to find the optimal value points; the gray dots in Fig. 7 show the % of revenue increase compared to the NM scenario.

The exponential equation keeps a simple interpretation for the optimal % of business customers to allocate by not allowing negative values. We use the optimal values points to fit them to a set of exponential functions. These functions help the hotel administrator to accept or reject customers.

The **upper model** considers only the maximum value point of each capacity; this model intends to get the maximum revenue increase, however, it may over-reject customers. On the other hand, the **lower model** considers only minimum optimal points of each capacity; this model fixes the over-rejection by accepting more tourist customers, as a result, it sacrifices a portion of the revenue improvement. The **central model** considers all the optimal values, and it offers a solution in the middle; Our recommendation is to with the central model.

Fig. 7 shows a set of exponential functions that adjust to optimal values; the solid line is the central model; the dashed lines represent the upper and lower models and they use only the maximum and the minimum data points. The dotted lines represent the "central models" under the scenarios where the demand increases/decreases $\pm$ 30%; we recreated the simulation and optimization process by changing demand parameters $\pm$ 30%. The equations for each of the models are in Table VI.

TABLE VI: PROPOSAL MODELS FUNCTIONS

Model	% of business customers	Data points used
Central	$0.8936e^{(-0.052)Cap}$	All
Upper	$1.4643e^{(-0.062)Cap}$	Max of each cap
Lower	$0.5036e^{(-0.058)Cap}$	Min of each cap
30%	$0.9397e^{(-0.046)Cap}$	All, w/+30% demand
-30%	$0.8067e^{(-0.072)Cap}$	All, w/ -30% demand

E. *Proposal's Validation*

We consider each of the equations in Table VI a model that can be used to determine the optimal number of rooms for business customers in a small hotel with a given capacity. Let's consider a small hotel with a total of 15 rooms. The hotel administrator can use the central model ($0.8936e^{(-0.052)(15)} = 13$) and allocate 13 rooms for the business customers for the season; if the administrator thinks the demand may vary $\pm$ 30%, he can use the +30% model to allocate 13 or the -30% model to allocate 11 rooms for business customers. On the other hand, if the administrator wants to cover any increased fluctuation over the last 10 years, he can use the upper model to allocate all capacity for business; or the lower model for any decreased fluctuation to allocate 7 rooms for business customers.

We want to evaluate how good (or bad) the models are. We validate the performance of the models using each of the simulations for capacities between 5-30 rooms. We test each of the models proposed using the ten years simulation data set. We compare the average revenue improvement of the models versus the No Model (NM) scenario versus the optimal values. We additionally measure the % of times the models are better than the NM scenario, the average of % of revenue when the models are above and below the NM scenario. The numerical results are in Table VII.

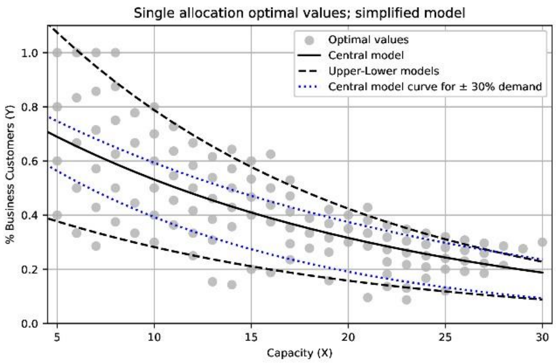

Fig. 7. We used the results of the ten simulated years to find the optimal % of business customers for each capacity (optimal data points). The solid line is the central model (optimal function) using all data points; the dashed lines represent the upper and lower models using only maximum and minimum data point for each capacity; the dotted lines is the central model (optimal functions) when the customer demand increases or decreased ± 30%. Using these models, we build a set of business rules.

Let's analyze the central, upper, and lower models. The upper model is the closest to the optimal values providing an average 3.2% of improvement, but only when it is above the NM (65% of the times). This contrasts to the lower model that offers more certainty to the result by improving the revenue 1.6%, 90% of the times. The lower model is recommended for risk-averse hotel administrators. The central model provides intermediate results between the upper and lower model by increasing 2.9% the revenue, 85% of the times.

Fig. 8 provides a visualization of the result by each of the capacities in the evaluation. The gray dots represent the maximum % of revenue (optimal) from the calibration process; the black dots represent the results of implementing the model; a good performing model is when gray and black dots are closer; a bad one is when dots are separate. Negative values imply that the model is not better than the NM scenario.

The customer's arrivals vary from season to season. For this reason, we also evaluate how robust the central model to ± 30% demand change. When the demand increases, the central model keeps improving the revenue by 3.5%, 93% of the times. Therefore, the central model is robust to demand increases, and the hotel administrator can keep using the central model without any adjustment. On the other hand, when the demand decreases, the central model still improves the revenue by 1.6% but only 65% of the times. We can validate these conclusions by observing the fourth graph in Fig. 8. The central model keeps improving the revenue for capacities lower than 12; from that point onward, the recommendation is to adjust the rejection rule to model "-30%" from Table VI.

The results provide a set of functions to determine the optimal strategy. The strategy adapts to demand variation and hotel administrator's risk-aversion level. The collection of models proposed improves the revenue, is simple to implement, and the hotel administrator can move quickly from one to another according to demand variations.

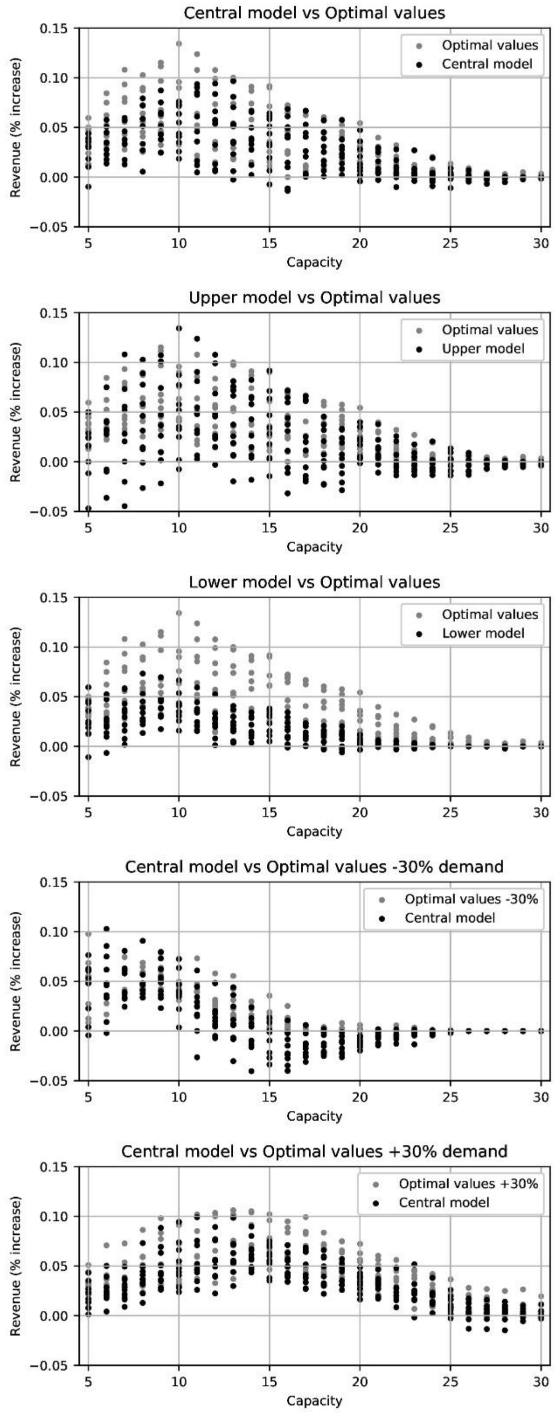

Fig. 8. We compare the performance of models using the simulated data. The gray dots represent the maximum % of revenue (optimal) from the optimization process; the black dots represent the results of implementing the model; a model's good performance is when gray and black dots are closer; a bad performance is when dots are separate. Negative values imply that the proposed model is not better than a No Model implementation.

TABLE VII: REVENUE PERFORMANCE OF OPTIMAL VALUES AND PROPOSAL MODELS VERSUS NO MODEL (NM)

Cap 5-30	Central	Upper	Lower	-30%	+30%
Optimal	3.20%	3.20%	3.20%	1.80%	4.20%
Proposal	2.40%	1.70%	1.40%	0.90%	3.20%
% times is above NM	85%	65%	90%	65%	93%
When above	2.90%	3.20%	1.60%	2.00%	3.50%
When below	-0.40%	-1.10%	-0.20%	-1.30%	0.40%

VI. Conclusions

The hotel sector is one of the few industries where small participants still play an essential role, and small hotels keep growing, [28]. These numbers highlight the importance for the small hotel administrators to continue innovating and keep maintain market share.

We confirmed that it is possible to implement RM models from the airline industry in a small hotel system. We implemented the single/double allocation model [22] and the bid pricing model from [2], (initially proposed for airlines) in a small hotel system. Data availability in the sector is highly restricted, the booking data is considered part of the business know-how and is not typically shared. We got high-level parameters of a small hotel in a US city during the summer period from a travel agency; we use agency parameters to simulate the client attendance, similar to [25]. Small hotels divide the customer into two types: business and tourist, commonly differentiated by the median of the days of booking in advance.

We proposed a method to calibrate the Capacity Allocation (single-double) and the resource bid prices model for two capacity scenarios: 10 and 20 rooms. We compare the RM models using the optimal parameters to the No Model (NM) implementation scenario; all the Revenue Models (RM) in this study increased revenue. The results show that the impact of the revenue management model is higher when the capacity is lower.

As a reference, we compared our results to other authors with similar RM models in the hotel sector. The % average improvement across all the papers is 9.9%. Although we cannot make a direct comparison, the number seems to be equivalent to the benefits reported in the Airlines (10% improvement year over year), [26]. There is an opportunity in the small hotel sector for adopting RM models.

S. E. Kimes [16] suggests that the hotel sector will be using RM models shortly, the same as the Airlines Industry did. This projection may be a risk for small hotels if they do not quickly adopt these techniques. The trend on Revenue Management models is moving to optimization self-adjusting systems focusing on large hotel chains. We want to facilitate the usage of RM models in small hotels considering their limited resources.

The single allocation model improved the revenue in all the capacity scenarios we studied. Although the single allocation model is not the best all the time, it is the easiest to optimize. The process consists of iterating the number of rooms allocated for business customers; for these reasons, we considered that this model is the most suitable to be implemented in a small hotel.

The results provided a set of functions to determine an optimal decision rule for accepting or rejecting the customer demand in a small hotel. The strategy adapts to demand variation and hotel administrator's risk-aversion level. The collection of models proposed improves the revenue, is simple to implement, and the hotel administrator can move quickly from one to another according to demand variations. We know each city has its own customers demand, but the small hotels have historical data for at least ten years (related to fiscal obligations). Therefore, any small hotel administrator can use the historical demand and iterates the number of rooms to allocate for the business customers to find the optimal values to fit them to exponential functions and determine their own business rules. Using the methodology proposed in this paper is much simpler than implementing some heuristic or machine learning RM models in the small hotel sector.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

The authors confirm contribution to the paper as follows: Jose-Mario Zarate-Carbajal contributed with the study conception and the implementation of the revenue management models. Riemann Ruiz-Cruz contributed with the design and implementation of the client attendance simulation. Juan-Diego Sanchez-Torres calibrated the models to find the optimal parameters. All authors equally contributed to analyze the results and propose the generalized model. All authors had approved the final version.

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