Research on Evolution of Digital Economy Ecosystem Based on Two-Dimensional Lotka-Volterra Model

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***Abstract*—From the perspective of ecology, we used ecosystem theory, symbiosis theory and evolutionary game theory to study the composition and characteristics of digital economic ecosystem and analyze the operation mechanism of digital economic ecosystem in this paper. Inspired by the complex system theory, we used Lotka-Volterra model to analyze the symbiotic evolution between the central enterprises and satellite digital enterprises in the ecosystem of digital economy.**

***Keywords*—digital economy, ecosystem, symbiotic evolution**

I. Introduction

In the new development stage, the digital economy is gradually becoming a key means of factor allocation and an important engine for high-quality development of the Chinese economy. In recent years, as a completely new form of economy, the digital economy has continuously changed the supply form of production factors, providing new paths and mechanisms for factor integration and allocation. The emergence of the digital economy fundamentally breaks the shackles of traditional resource allocation methods. With powerful resource integration capabilities, digital technology reshapes the traditional production factor system and achieves factor allocation upgrading. The digital economy empowers market players to undergo digital transformation, promotes traditional capital, labor, and other production factors to achieve multi-channel allocation and improve resource allocation. The scale of China's digital economy has ranked second in the world. In 2022, the scale of China's digital economy reached 50.2 trillion yuan, ranking second in the world in total, with a year-on-year nominal growth of 10.3% and a proportion of 41.5% in the gross domestic product. It is expected that by 2035, the scale of China's digital economy will reach over 110 trillion yuan, with a penetration rate of 48% and an employment capacity of 4 1.5 billion. However, the problem of resource mismatches caused by the extensive factor input-driven development model and the lagging development of factor markets has become a key factor restricting the high-quality development of the Chinese economy. With the reality of diminishing marginal output of capital and labor and the disappearance of China's demographic dividend, achieving the reorganization and upgrading of stock factors, effective allocation, and establishing an efficient and unified market system to improve resource mismatch levels plays a key role in improving total factor productivity and enhancing the quality and efficiency of economic development.

II. Literature Review

The next subsections provide instructions on how to insert figures, tables, and equations in your document. In the early days, the digital economy mainly revolved around the promotion of economic development. Tapscott (1996) first put forward the concept of Digital Economy, and predicted that digital economy is a new economy, new enterprise, new technology, and the effective interaction among them. In recent years, more and more scholars pay attention to the new trend of digital economy from the perspective of ecology, among which the business ecosystem and innovation ecosystem are the two most widely used fields of ecology and complex system theory. In addition to its application in the economic field (Rong, 2015; Faissal, 2018; Yusaf, 2022; Ying, 2023; Uddin, 2023; Xiong; 2023; Tao, 2023; Rong, 2023; Ren, 2023), digital economy has also been widely used in other fields. Fields such as finance (Liu, 2023; Huo, 2023; Yu, 2023; Laurence, 2023); business (Stenroos, 2017; Xie, 2020); production line (Grosse, 2018; Ding, 2018); and so on (Chijindu, 2023).

The core theory of the ecosystem holds that in a given time and space, all kinds of biological communities interact with their abiotic environment in the process of energy exchange, material sharing and information transmission, which forms a dynamic complex integrated system with regular interaction, coexistence and death, and loose and open characteristics. According to the basic idea of ecosystem, Moore (1996) first put forward the concept of the community of shared destiny of commercial ecosystem, he thought that the interrelated economic organization will form the dynamic structural system. The ecosystem makes use of the heterogeneity of each complementary enterprise in the strategic reconfiguration of the core enterprise, and the system will go through the stages of startup, expansion, maturity, decline or revival and upgrading (Han, 2020), and have the value co-created, co-shared and co-taken in the evolutionary process (Ritala, 2013).

Currently, many scholars have studied enterprise ecosystem, regional innovation ecosystem and national innovation ecosystem. However, few scholars have analyzed the development characteristics and operation mechanism of digital economy from the perspective of ecology. Meng (2020) used Lotka-Volterra model to explore the symbiotic stability of the central enterprise population and the satellite digital enterprise population within the digital economy ecosystem, and analyzed the evolution process of the digital

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economy ecosystem, but the stability conditions of the equilibrium point given by them are not correct.

Based on the characteristics of digital economy as a complex and dynamic ecosystem, this paper aims to analyze the composition and characteristics of the system by using the theory of differential dynamics and explore the stability of the operation and symbiotic mechanism of the digital economy ecosystem. This paper will correct the calculation error made by Meng Fanglin and others, and give the correct conditions for the stability of the equilibrium point of the system.

III. Model Analysis

The dual population Lotka-Volterra model is an extremely widely used two-dimensional differential dynamics system, which can accurately describe the satellite like competitive symbiotic relationship between the central enterprise and the digital population, and determine the impact of the core population in the entire ecosystem evolution process. In this section we adopts this low-dimensional dynamic system to simulate the dynamic relationship between central enterprises and other digital enterprise populations within the digital economy ecosystem. The language of differential dynamical systems can be used to characterize the relationship between the population of central core digital enterprises and the population of satellite digital enterprises as follows:

$$\begin{aligned} \frac{dx_1}{dt} &= r_1 x_1\left[1-\frac{x_1}{K_1}+a_{21}\frac{x_2}{K_2}\right] \\ \frac{dx_2}{dt} &= r_2 x_2\left[1+a_{12}\frac{x_1}{K_1}-\frac{x_2}{K_2}\right] \end{aligned} \tag{1}$$

where $x_1(t)$ and $x_2(t)$ represent the output of the population of central enterprises and satellite digital enterprises over time, respectively; K_1 represents the maximum contribution rate and influence coefficient of the central enterprise to other digital enterprises, while K_2 represents the maximum contribution rate of other digital enterprises to the central enterprise. Different values of K_1 and K_2 can represent the different symbiotic evolution patterns between the satellite type central enterprise population and other digital enterprises in the digital economy ecosystem.

Frist of all, we will discuss the stability of the equilibrium point of system (1). Because the stable state of the evolutionary equilibrium points of the central enterprise population and satellite digital enterprise population in the digital economy ecosystem has several completely different forms. For example, if the equilibrium point is in a stable state, then the equilibrium point of the system indicates that both parties have reached the maximum output and maintained stability.

Let

$$\frac{dx_1}{dt}=0, \quad \frac{dx_2}{dt}=0$$

We can easily obtain the four equilibrium points of system (1) by solving the corresponding linear equation system based on the right side of system (1) being zero:

$E_1(0,0)$, $E_2(K_1,0)$, $E_3(0, K_2)$, $E_4\left(\frac{K_1(1+a_{21})}{1-a_{12}a_{21}}, \frac{K_2(1+a_{12})}{1-a_{12}a_{21}}\right)$

According to the qualitative theory of differential equations, as long as the determinant and trace symbols of the Jacobian matrix are calculated and judged, the stable equilibrium point and stability conditions of the symbiotic evolution model of the digital economy ecosystem can be obtained easily.

Theorem1:

(i) If $r_1 > 0$, $r_2 > 0$, then equilibrium point $E_1(0,0)$ is unstable.

(ii) If $a_{12} < -1$, then equilibrium point $E_2(K_1,0)$ is stable.

(iii) If $a_{21} < -1$, then equilibrium point $E_3(0, K_2)$ is stable.

(iv) If $a_{12} > -1$, $a_{21} > -1$, and $a_{12}a_{21} < 1$, then equilibrium point $E_4\left(\frac{K_1(1+a_{21})}{1-a_{12}a_{21}}, \frac{K_2(1+a_{12})}{1-a_{12}a_{21}}\right)$ is stable.

Proof:

(i) Notice that the Jacobian matrix at the equilibrium point $E_1(0,0)$ is:

$$J(E_1)=\begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}$$

Hence, it is clear that:

$$Tr(E_1)= r_1+r_2 > 0;$$

$$Det(E_1)= r_1 r_2 > 0$$

According to the ordinary differential qualitative theory, we know that equilibrium point $E_1(0,0)$ is unstable.

(ii) It is easy to obtain that the Jacobian matrix at the equilibrium point $E_2(K_1,0)$ is:

$$J(E_2)=\begin{pmatrix} -r_1 & \frac{a_{21}r_1}{k_2}\cdot r_2 \\ 0 & r_2(1+a_{12}) \end{pmatrix}$$

Therefore, when $a_{12}<-1$, we have the trace and determinant about matrix J as follows:

$$Tr(E_2)= -r_1+r_2(1+a_{12}) < 0;$$

$$Det(E_2)= -r_1 r_2(1+a_{12}) > 0.$$

According to the ordinary differential qualitative theory, the equilibrium point $E_2(K_1,0)$ is stable.

(iii) It is easy to obtain that the Jacobian matrix at the equilibrium point $E_3(0, K_2)$ is:

$$J(E_3)=\begin{pmatrix} r_1(1+a_{21}) & 0 \\ \frac{a_{12}r_2}{k_1}\cdot k_2 & -r_2 \end{pmatrix}$$

Therefore, when $a_{21}<-1$, we have the trace and determinant about matrix J as follows:

$$Tr(E_3)= -r_1+r_2(1+a_{12}) < 0;$$

$$Det(E_3)= -r_1 r_2(1+a_{12}) > 0.$$

According to the ordinary differential qualitative theory, we get that equilibrium point $E_3(0, K_2)$ is stable.

(iv) It is easy to obtain that the Jacobian matrix at the equilibrium point E_4 is:

$$J(E_4)=\begin{pmatrix} -\dfrac{r_1(1+a_{21})}{1-a_{12}a_{21}} & \dfrac{r_1a_{21}(1+a_{21})}{1-a_{12}a_{21}}\cdot\dfrac{k_1}{k_2} \\ \dfrac{r_2a_{12}(1+a_{12})}{1-a_{12}a_{21}}\cdot\dfrac{k_2}{k_1} & -\dfrac{r_2(1+a_{12})}{1-a_{12}a_{21}} \end{pmatrix}$$

Therefore, when $a_{12}>-1$, $a_{21}>-1$, and $a_{12}a_{21}<1$, we have the trace and determinant about matrix J as follows:

$$Tr(E_4)=-\frac{r_1(1+a_{21})+r_2(1+a_{12})}{1-a_{12}a_{21}}<0;$$

$$Det(E_4)=\frac{r_1r_2(1+a_{21})(1+a_{12})}{1-a_{12}a_{21}}>0.$$

According to the ordinary differential qualitative theory, we get that equilibrium point E_4 is stable.

Theorem1 means that (i) The inequality $r_1>0$, $r_2>0$ holds all the time, which means that the intrinsic growth rate of the central enterprise population and other digital enterprises are both positive. Therefore, E(0,0) is an unstable equilibrium point, which means that the number of central enterprise populations and other digital enterprise populations has been increasing positively, and neither population is likely to go extinct.

(ii) When $a_{12}<-1$, in this situation, the central enterprise will have a negative impact on other digital enterprises, which will inevitably restrain the growth of the population of other digital enterprises. Ultimately, the central enterprise will reach a monopoly position in the market, causing the population of other digital enterprises to approach zero, and the central enterprise population will reach its maximum capacity K_1.

(iii)When $a_{21}<-1$, at this moment, a_{21} is a negative number, it means that other digital enterprises have an inhibitory effect on the central enterprise, resulting in a downward trend in the number of central enterprise populations, while other enterprise populations reach their maximum capacity K_2.

(iv)When $-1<a_{12}<0$, $-1<a_{21}<0$, and $a_{12}a_{21}<1$. Under these conditions, the stability of E_4 is of practical significance. On the phase plane, E_4 is an asymptotically stable node or an asymptotically stable degenerate node. This situation describes how the two populations (the central enterprise and other digital enterprises) will approach the equilibrium point E_4, indicating that the central enterprise will gradually maintain its population's saturation level, i.e. $\frac{K_1(1+a_{21})}{1-a_{12}a_{21}}$, meanwhile, $\frac{K_2(1+a_{12})}{1-a_{12}a_{21}}$ is the population's saturation level for other digital enterprises.

Theorem2 If $a_{12}a_{21}=1$, Then system (1) has general integral as follows for arbitrary constant C:

$$x^{\alpha}y^{\beta}=Ce^{\rho_1 t} \qquad (2)$$

where $\beta=\frac{r_1}{r_2}\cdot a_{21}\alpha$,

$\rho_1=r_1(1+a_{21})\alpha$.

Proof: For the two undetermined coefficients α and β, we consider the differential expression derived from system (1) as follows:

$$\alpha\cdot\frac{\dot{x}}{x}+\beta\cdot\frac{\dot{y}}{y}=(r_1\alpha+r_2\beta)+\left(-\frac{r_1}{K_1}\alpha+\frac{r_2a_{12}}{K_1}\beta\right)x+\left(\frac{r_1a_{21}}{K_2}\alpha-\frac{r_2}{K_2}\beta\right)y \qquad (3)$$

Let:

$$-\frac{r_1}{K_1}\alpha+\frac{r_2a_{12}}{K_1}\beta\alpha=0$$

$$\frac{r_1a_{21}}{K_2}\alpha-\frac{r_2}{K_2}\beta=0$$

If condition $a_{12}a_{21}=1$ be met, then the above homogeneous linear equation sets has non-zero solutions. We select α as an arbitrary constant, then from above homogeneous linear equation sets we have:

$$\beta=\frac{r_1}{r_2}\cdot a_{21}\alpha$$

$$\rho_1=r_1(1+a_{21})\alpha$$

Therefore, dynamic system (2) can be rewritten as following form:

$$\alpha\cdot\frac{\dot{x}}{x}+\beta\cdot\frac{\dot{y}}{y}=\rho_1 \qquad (4)$$

The conclusion of theorem 2 can be obtained by integrating t from zero to both sides of (4).

Theorem 2 means that in the three dimensional space (x,y,t), the motion trajectory of ecosystem (1) must be on the spatial surface (2). When $\rho_1>0$, at least one of the populations of the central enterprise and other digital enterprises infinitely approaches the upper limit of their population; When $\rho_1<0$, at least one of the populations of central enterprises and other digital enterprises approaches extinction infinitely.

In the same way, we can obtain the following results:

Theorem3 $a_{12}a_{21}\neq 1$, $y=k_1+k_2x$ ($r_1a_{21}\neq r_2$), Then system (1) has general integral as follows:

$$x^{\alpha}y^{\beta}=Ce^{\rho_2 t} \qquad (5)$$

where

$$\alpha=\frac{r_2}{1-a_{12}a_{21}},$$

$$\beta=\frac{r_1a_{21}}{1-a_{12}a_{21}},$$

$$\rho_2=\frac{r_1r_2(1+a_{21})}{1-a_{12}a_{21}},$$

$$k_1=-\frac{K_2}{a_{21}},$$

$$k_2=\frac{K_2}{K_1}\cdot\frac{r_1-r_2a_{12}}{r_1a_{21}-r_2}.$$

Theorem 3 means that in the three dimensional space (x,y,t), the motion trajectory of ecosystem (1) is a family of spatial curves formed by the intersection of a spatial surface $x^{\alpha} y^{\beta} = Ce^{\rho_2 t}$ and a spatial plane $y = k_1 + k_2 x$ (). $r_1 a_{21} \neq r_2$ When $\rho_2 > 0$, At least one of the two populations we are discussing tends towards its upper limit of quantity; When $\rho_2 < 0$, at least one of the two populations tends to become extinct.

IV. Conclusion

From a long-term perspective, it is crucial to view the development of the digital economy, resource mismatch, and a unified market. The development of the digital economy is also a process of correcting resource mismatch and promoting optimal allocation of factors, and the process of marketization is constantly accelerating. Therefore, it is crucial to treat the three as a complete “ecosystem”. The market plays a decisive role in resource allocation. Before the arrival of the digital economy, the effective flow of spatial elements in different regions was poor, and the efficiency of market resource allocation was difficult to fully demonstrate. The issue of regional barriers makes it difficult for regions with excessive resource allocation to transport and transfer to regions with insufficient resource allocation. At this time, the advantages of the spatiotemporal interconnection of the digital economy will be fully demonstrated. In the era of digital economy 3.0, the openness and sharing of data, the dynamic nonlinearity of business processes, the innovative growth of enterprises, and the diverse coexistence of entities have endowed the digital economy with ecosystem characteristics, making it possible for massive entities to efficiently gather and collaborate in different regions. The digital economy is giving birth to and aggregating various innovative resources, injecting new momentum into the economic development of various countries.

Digital economy ecosystem has a self-incubation function. If its derivative superposition effect can be fully utilized, the system can dynamically expand and evolve through endogenous tillers and exogenous derivation. The digital economies of Japan and China are both on the track of rapid development. In 2022, the digital economy scale of five major world countries, including the United States, China, Germany, Japan, and South Korea, have reached $31 trillion dollars. In terms of overall size, Japan’s digital economy stood at $256.9 billion in 2021, ranking fourth in the world in terms of the size of the digital economy, with the top three being the United States, China and Germany. Based on the cultural background and highly complementary technology, the two countries should have a brilliant future in digital cooperation. Having bright prospects for future cooperation in the field of digital economy, China and Japan have the possibility to make concerted effort to build a more harmonious digital enterprise ecosystem, promote cooperation on new technologies, new products, new models and new forms of business, and bring the world more benefit with the digital civilization of Asia.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Wang Yiruo determined the theme and the selected the dynamic methods of this article; Lv Mingxia used ecosystem theory to analyze the composition and characteristics of digital economic ecosystem and analyze the operation mechanism of digital economic ecosystem in this paper. By using Lotka-Volterra model to analyze the symbiotic evolution between the central enterprises and satellite digital enterprises in the ecosystem of digital economy. Ji Ping was responsible for the 2021 planning project of the Shanghai Higher Education Association, titled “Practical Research on the Transformation and Upgrade of Traditional Business Majors in the Context of the Digital Economy” (Approval No. Y2-52). The problems encountered in this project provide practical basis and financial support for the theoretical development of this study; Ying Yirong inspected the proof process of all theorems in the paper to ensure the correctness of the theorems; all authors had approved the final version.

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References

Aarikka-Stenroos, L., Ritala, P. 2017. Network management in the era of ecosystems: systematic review and management framework. *Ind. Mark. Manag*. 67: 23–36.

Chijindu, I. O. 2023. Research on the digital economy: Developing trends and future directions. *Technological Forecasting & Social Change*, 193, 122635.

Da, H., Zhang, X. T., Di, R. Q. 2022. Green finance and energy efficiency: Dynamic study of the spatial externality of institutional support in a digital economy by using hidden Markov chain. ***Energy Economics***, 116, 106431.

Ding, L., Wu, J. 2018. Innovation ecosystem of CNG vehicles: A case study of its cultivation and characteristics in Sichuan, China. ***Sustain***. 10: 1–16.

Faissal, B. N., Armellini, F. 2018. Systems of innovation and innovation ecosystems: a literature review in search of complementarities. ***J. Evol. Econ***., 28, 1053–1080.

Grosse, M. 2018. How user-innovators pave the way for a sustainable energy future: A study among German energy enthusiasts. ***Sustain***., 10: 1–16.

Han, J., Wang, Y. M., Tu, Y. H. 2020. The ecosystem concept in strategic management research: An integrated dynamic process model. ***Science & Technology Progress and Policy***, 37(1): 1-9.

Laurence, L. D. 2023. Fintech RE in a global finance centre: Expert perceptions of the benefits of and challenges to digital financing of distributed and decentralised renewables in Hong Kong. ***Energy Research & Social Science***, 97,102997.

Liu, Y. T., Zheng, M. B., Wai, Y. S. 2023. On the linkages between digital finance and real economy in China: A cointegration analysis.

Innovation and Green Development, 80, 102081.

Md Riaj, U. 2023. The role of the digital economy in Bangladesh's economic development. ***Sustainable Technology and Entrepreneurship***, 3, 100054.

Meng, F. L. 2020. Research on operation mechanism and evolution of digital economy ecosystem based on Lotka-Volterra model. ***Journal of Hehai University*** **(*Philosophy and Social Sciences Edition*)**, 22(2): 63-71, 2020.

Moore, J. F. 1996. *The Death of Competition: Leadership & Strategy in the Age of Business Ecosystems*, U.S.A.: Harper Business, pp. 82-94

Ren, J. Y., Xu, Z. T., Hu, M. Z. 2023. Digital economy and settlement intention of migrants in urban China. ***Telecommunications Policy***, 47, 102666.

Ritala, P., Agouridas, V, Assimakopoulos, D. 2013. Value creation and capture mechanisms in innovation ecosystems: A comparative case study. ***International Journal of Technology Management***, 63(3-4): 244-267.

Rong, K., Hu, G., Lin, Y., Shi, Y., Guo, L. 2015. Understanding business ecosystem using a 6C framework in internet-of-things-based sectors. ***Int. J. Prod. Econ.*** 159: 41–55.

Rong, K., Luo, Y. N. 2023. Toward born sharing: The sharing economy evolution enabled by the digital ecosystems. ***Technological Forecasting and Social Change***, 196, 122776.

Tapscott, D. 2014. ***The digital economy anniversary edition: Rethinking promise and peril in the age of networked intelligence***, U.S.A.: McGraw-Hill, pp. 1-78.

Tao, M. M., Stephen, P., Zheng, Y. H. 2023. Appraising the role of the digital economy in global decarbonization: A spatial non-linear perspective on globalization. ***Journal of Environmental Management***, 347, 119170.

Wang, Y., Wang, Y., Muhammad, S. 2023. How does digital economy affect energy poverty? Analysis from the global perspective. *Energy*, 282, 128692.

Xie, X., Wang, H. 2020. How can open innovation ecosystem modes push product innovation forward? An fsQCA analysis. ***J. Bus. Res.***, 108: 29–41.

Xiong, M. Z., Zhang, F., Wang, H. Z. 2023. Digital economy, credit expansion, and modernization of industrial structure in China. ***Finance Research Letters***, 58, 104500.

Yusaf, H. A., Andrea, T. 2022. The digital economy and the growth dynamics of sharing platforms: A transaction cost economics assessment. *Journal of Digital Economy*, 1(3): 209-226.

Yu, H. Y., Wei, W., Ying, L. 2022. The impact of green digital finance on energy resources and climate change mitigation in carbon neutrality: Case of 60 economies. ***Resources Policy***, 79, 103116.